國立臺灣科技大學
九十一學年度碩士班招生考試試題

系所組別： 電機工程系乙一組
科　目： 線性代數

※總分100分

1. Verify that the equation of a straight line through the distinct points (x_1, y_1) and (x_2, y_2) is $\begin{vmatrix} x & y & 1 \\ x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \end{vmatrix} = 0$. (12%)

2. Determine all possible values of λ such that the following homogeneous systems admit nontrivial solutions.

(a) $\begin{bmatrix} 2-\lambda & 1 \\ 1 & 2-\lambda \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = 0$ and (b) $\begin{bmatrix} 5 & \lambda \\ 2 & -4 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = 0$ (14%)

3. Prove that a square matrix A is invertible if and only if its columns (or rows) are linearly independent. (12%)

4. For the case of rotation a plane, the transformation Q corresponds a counterclockwise rotation of the basis through an angle θ. Does Q^{-1} correspond to the reverse of this, a clockwise rotation θ? ***Prove or disprove your conclusion.*** (12%)

5. Use the augment matrix and elimination method to find the inverse of

$A = \begin{bmatrix} 1 & 4 & 7 \\ 2 & 5 & 8 \\ 3 & 6 & 10 \end{bmatrix}$, $A^{-1} = ?$ (10%)

(Note: there are no score for using other methods)

6. Find the solution set for the following linear matrix equation

$$Ax = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 2 & 2 & 0 & 3 \\ 0 & 4 & -4 & 5 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \\ -7 \end{bmatrix}.$$ (15%)

7. Let $A = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix}$, find $A^{10} = ?$ (15%)

8. Let the linear transform $T: R^2 \to R^3$ is given by

$T(u_1) = v_1 + v_2 + v_3$ and $T(u_2) = v_1 - v_2$,

where $u_1 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$, $u_2 = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$, and $v_1 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$, $v_2 = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$, $v_3 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$. Find the standard transformation matrix A for linear transform T. (10%)