

國立臺灣科技大學

九十一學年度碩士班招生考試試題

系所組別：電機工程系丙一組、電機工程系丙二組

科目：線性代數與機率

(共八題；滿分一百分)

1.(10%) Let $A = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 2 \\ 2 & -1 & 0 \\ 1 & -1 & -1 \end{bmatrix}$. Please find a basis of all x 's that satisfy

$Ax=0$. Also find a basis of all b 's for which $Ax=b$ has feasible solutions.

2.(10%) Let H be a subspace formed by $x_1 = [1 \ 0 \ 1 \ 0]^T$, $x_2 = [2 \ 1 \ 2 \ 1]^T$, and $x_3 = [-1 \ 2 \ -3 \ 4]^T$. Please find an orthonormal basis for H .

3.(10%) Let x and y be two eigenvectors of a real symmetric matrix corresponding to distinct eigenvalues. Show that x and y are orthogonal.

4.(10%) Let $A = \begin{bmatrix} 3 & -1 & 1 \\ -2 & 1 & 1 \\ 1 & -1 & -2 \end{bmatrix}$. Find the determinant of A and A^{-1} .

5.(10%) Let $A = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}$. Find a matrix Y such that $Y^T A Y$ is a

diagonal matrix.



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6. (10%) Let X be a discrete random variable uniformly distributed in the set $S = \{1, 2, \dots, n\}$. Let $Y = K + LX$, where K and L are integers. Find the mean and variance of Y .
7. (15%) Let $Y = aX + b$, where a and b are constants.
- (a) (5%) Find the characteristic function of Y in terms of the characteristic function of X .
- (b) (10%) Find the characteristic function of Y if X is a zero-mean, unit-variance Gaussian random variable.
8. (25%) The random vector variable (X, Y) has the joint pdf
- $$f(x, y) = k(x + y) \quad 0 < x < 1, 0 < y < 1.$$
- (a) (5%) Find k .
- (b) (5%) Find the joint cdf of (X, Y) .
- (c) (5%) Find the marginal pdf of X and of Y .
- (d) (5%) Find $f_Y(y|x)$.
- (e) (5%) Are X and Y independent? Why?

Formula that you may use:

$$\sum_{i=1}^n i = \frac{n(n+1)}{2},$$

$$\sum_{i=1}^n i^2 = \frac{n(n+1)(2n+1)}{6}$$

