國立臺灣科技大學
九十一學年度碩士班招生考試試題
系所組別：資訊工程系、電子工程系甲組、電機工程系丙一組
科　目：資料結構

總分 100 分

1. 10% Suppose there are n internal nodes of a binary tree.
(a) 2% How many external nodes of this tree?
(b) 2% Let E and I denote the external path length and internal path length of a binary tree, respectively. How to relate E to I?
(c) 3% Based on (b), what is the smallest value of I?
(d) 3% Based on (b), what is the largest value of I?
2. 20% A binary search tree contains the identifiers a_1, a_2, ..., a_n with $a_1 < a_2 < ... < a_n$ and the probability of searching for each a_i is p_i then the total cost of any binary search tree is: $\sum_{i=1}^{n} p_i level(a_i)$. As for the unsuccessful searches, we may partition the identifiers that are not in the binary search tree into n+1 classes E_i, $0 \leqq i \leqq n$. E_0 contains all identifiers x such that $x < a_1$. E_i contains all identifiers x such that $a_i < x < a_{i+1}$, $1 \leqq i < n$, and E_n contains all identifiers x, $x > a_n$. Note that there is a failure node i corresponding to E_i. If q_i is the probability that the identifier we are searching is in E_i, then the cost of the failure nodes is: $\sum_{i=0}^{n} q_i(level(failure\ node\ i) - 1)$. Therefore, the total cost of a binary search tree is: $\sum_{i=1}^{n} p_i level(a_i) + \sum_{i=0}^{n} q_i(level(failure\ node\ i) - 1)$. An optimal binary search tree for the identifier set a_1, a_2, ..., a_n is one that minimizes the cost over all possible binary search trees for this identifier set. Let T_{ij} denote an optimal binary search tree for a_{i+1}, ..., a_j, $i<j$. T_{ii} is an empty tree for $0 \leqq i \leqq n$ and T_{ij} is not defined for $i>j$. Let c_{ij} denote the cost of the search tree T_{ij}. By definition c_{ii} is 0. Let r_{ij} denote the root of T_{ij} and let $w_{ij} = q_i + \sum_{k=i+1}^{j}(q_k + p_k)$ denote the weight of T_{ij}. By definition, $r_{ii}=0$ and $w_{ii}=q_i$, $0 \leqq i \leqq n$. T_{0n} is an optimal binary search tree for a_1, a_2, ..., a_n. Its cost is c_{0n}, its weight is w_{0n}, and its root is r_{0n}.
(a) 10% If T_{ij} is an optimal binary search tree for a_{i+1}, ..., a_j and r_{ij}=k, then k satisfies the inequality $i<k \leqq j$. T_{ij} has two subtrees L and R. L is the left subtree and contains the identifiers a_{i+1}, ..., a_{k-1} and R is the right subtree and contains the identifiers a_{k+1}, ..., a_j. Show that $c_{ij} = w_{ij} + c_{i,k-1} + c_{kj}$.
(b) 10% Let n=3 and (a_1, a_2, a_3) = (Disney, Florida, Orlando). Let (p_1, p_2, p_3) = (3, 3, 1) and (q_0, q_1, q_2, q_3) = (2, 3, 1, 1). (The p's and q's are multiplied by 14 for convenience). Find the optimal binary search tree for T_{03}. Initially $w_{ii}=q_i$, r_{ii}=0 and c_{ii}=0, $0 \leqq i \leqq 3$. (Note that all values

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of w, c and r should be displayed for each step instead of only the final optimal binary search tree shown.)

3. 10% A 2-3 tree is defined as each internal node of a 2-3 tree has degree two or three. A degree two node is called a 2-node while a degree three node is called a 3-node.

(a) 2% What is the number of elements in a 2-3 tree with height h (the external nodes are at level h+1 and the root is at level 1)?

(b) 4% Based on Figure 1, show the resulting tree after inserting 3 elements with keys 70, 30 and 60, respectively (the number shown in a node is the key of the element located in that node).

(c) 2% What is the time complexity of inserting an element into a 2-3 tree with n elements?

(d) 2% What is the time complexity of deleting an element from a 2-3 tree with n elements?

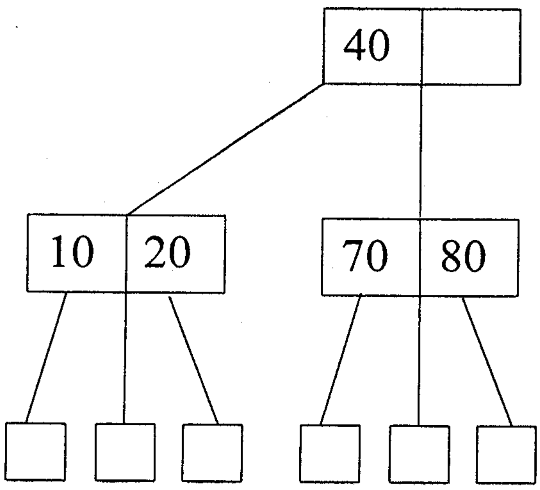

Figure 1: A 2-3 tree.

4. 10% A depth first spanning tree can be created after applying the depth first search algorithm to a connected undirected graph, G. The sequence numbers in which the vertices are visited during the depth first search are called the depth first numbers, or dfn, of the vertices. The root of a depth first spanning tree is an articulation point iff it has at least two children. In addition, any other vertex u is an articulation point iff it has at least one child w such that we cannot reach an ancestor of u using a path that consists of only w, descendants of w, and a single back edge. These observations lead us to define a value, low, for each vertex of G such that low(u) is the lowest depth first number that we can reach from u using a path of descendants followed by at most one back edge. A depth first spanning tree is created for Figure 2.(a) and it is shown in Figure 2.(b), where the number outside the vertex is the dfn number.

(a) 4% Based on Figure 2.(b), put the low number of each vertex in Figure 2.(c).

(b) 3% If u is an articulation point and it has a child w then what the relationship between low(w) and

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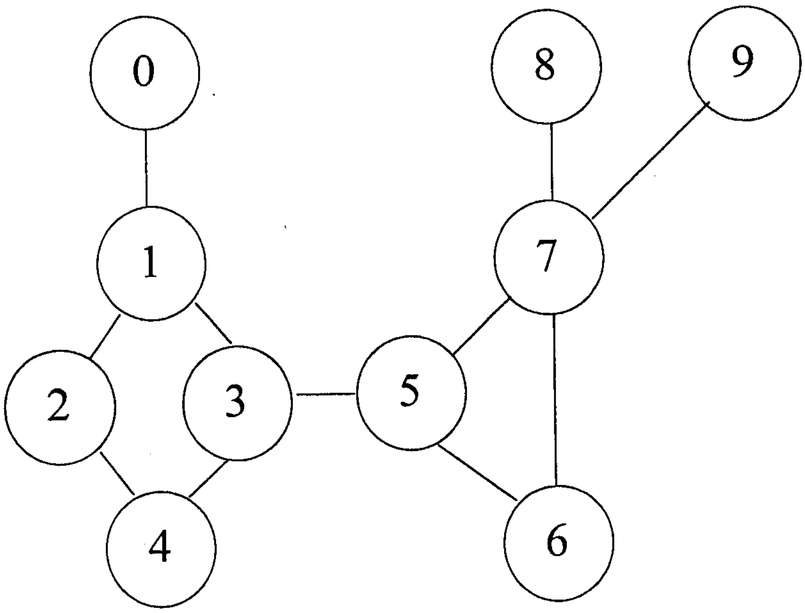
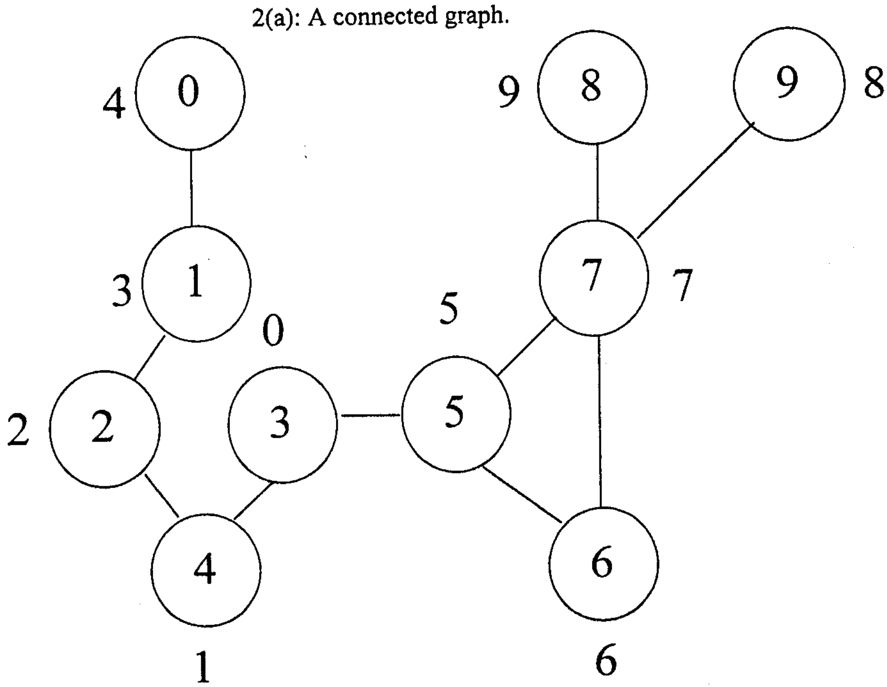
dfn(u) is.

(c) 3% Based on (a) and (b), find out all articulation points of G.



2(a): A connected graph.



2(b): A depth first spanning tree corresponding to (a).

Vertex	0	1	2	3	4	5	6	7	8	9
dfn	4	3	2	0	1	5	6	7	9	8
low										

2(c): dfn and low values.

Figure 2: A connected graph G.

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5. Assume that we have a forest of three trees as illustrated in Figure 3. Please transform the forest into its binary tree representation. (12%)

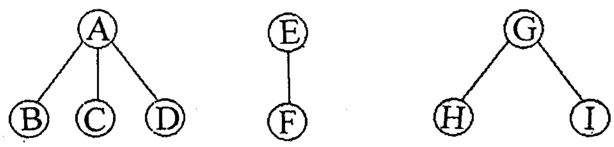

Figure 3

6. Assume that we want to represent trees using the list representation and assume that we define the node structure as:

tag = TRUE/FALSE	*dlink/data*	*link*

where the tag is a field that holds the value of TRUE if the node is a link node, and the value of FALSE if the node is a data node. Please represent the sample tree shown in Figure 4 using the list representation. (14%)

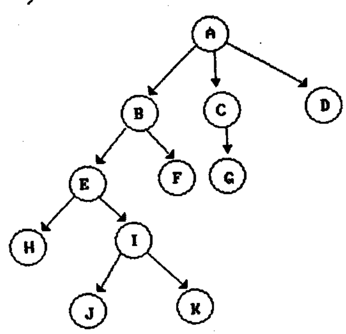

Figure 4

7. Assume that the input list is (26, 5, 77, 1, 61, 11, 59, 15, 48, 19). Please draw the merge tree of the input list using the iterative merge sort scheme. (12%)

8. Please write the postfix form of the following expression: (12%)

a / b – c + d * e – a * c