

國立臺灣科技大學

九十一學年度碩士班招生考試試題

系所組別：工業管理系甲組、乙組、丙組、丁組

科目：統計學

1. (30 points) (計算至小數點第 4 位)

Suppose a random chosen individual's verbal score X and quantitative score Y on a nationally administered aptitude examination have joint pdf

$$f(x, y) = \begin{cases} \frac{2}{5}(2x+3y) & 0 \leq x \leq 1, 0 \leq y \leq 1 \\ 0 & \text{otherwise} \end{cases}$$

- (5 points) Find the marginal distribution of X alone.
- (5 points) Find $\mu_X = E(X)$ and $\sigma_X^2 = E(X - \mu)^2$.
- (5 points) Using Chebyshev's theorem, find the interval that contains at least $5/9$ of the values of X .
- (5 points) For the interval of X obtained in (c), find the true probability.
- (10 points) You are asked to provide a prediction T of the individual's total score $X+Y$. The error of prediction is the mean squared error $E[(X+Y-T)^2]$. What value of T minimizes the error of prediction?

2. (20 points) (計算至小數點第 4 位)

Let X be the number of flaws on the surface of a randomly selected boiler of a certain type. Suppose X has a Poisson distribution with mean λ . However, the mean λ is unknown. Past experience indicates that the prior probabilities of λ are $P\{\lambda = 4\} = 0.3$, $P\{\lambda = 5\} = 0.5$ and $P\{\lambda = 6\} = 0.2$.

- (5 points) Given that the true mean of X is 5, what is the probability that a randomly selected boiler has at most two flaws?
- (5 points) Under the prior probabilities, what is the probability that a randomly selected boiler has exactly two flaws?
- (10 points) Suppose that a boiler was randomly selected and found that it had exactly two flaws. Using this information, revise the prior probabilities of λ .



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3. (25%)

調查 7 名新任職的大學生之在校成績(X)與起薪(Y)，(月薪，

單位：萬元)如下：

在校學業成績(X)	90.8	76	79.7	62.8	74.1	83	64.8
起薪(Y)	3.4	2.8	2.8	2.2	2.5	3.1	2.3

可計算出：

$$\sum_i X_i = 531.2, \sum_i Y_i = 19.1, \sum_i X_i^2 = 40895.42, \sum_i Y_i^2 = 53.23, \sum_i X_i Y_i = 1474.43$$

- (a) 求 X 、 Y 的樣本相關係數。
- (b) 求 Y 對 X 的直線迴歸方程式 $\hat{Y} = \hat{\beta}_0 + \hat{\beta}_1 X$ 。
- (c) 在顯著水準 $\alpha = 0.05$ 下，檢定 $H_0: \beta_1 = 0$ 對 $H_1: \beta_1 \neq 0$ 。
- (d) 求判定係數 R^2 。
- (e) 若某生在校學業成績 85 分，試求其起薪待遇的 95% 的預測區間。

4. (15%)

台灣體育協會針對一般民眾對球類的喜好情形作一調查，在北中南三區隨機抽出部份民眾作為訪問對象，調查結果如下表：

	北區		中區		南區		總數
	男	女	男	女	男	女	
棒球	90	110	60	80	50	70	460
羽球	50	30	40	40	54	46	260
籃球	70	50	44	36	40	40	280
樣本數	210	190	144	156	144	156	1000

試問：在顯著水準為 $\alpha = 0.05$ 之下，不同地區的民眾對於球類運動之喜好是否有顯著差異？

5. (10%)

某製造業的公司為了要瞭解哪一家廠商所供應的零件其品質較好，以便能夠決定到底要下訂單給甲廠商或是乙廠商。因此分別各從兩家供應商隨機抽取 16 個產品來作試驗，得到甲供應商的樣本平均數 $\bar{X} = 300$ ，樣本標準差 $S_1 = 15$ ，而乙供應商的樣本平均數 $\bar{Y} = 276$ ，樣本標準差 $S_2 = 18$ 。在假定全體分配近似於常態分配的情況下，試以 $\alpha = 0.05$ 為顯著水準，檢定兩家供應商所提供的產品品質是否有差異。(假設兩家廠商的變異數均相等)



國立臺灣科技大學

九十一學年度碩士班招生考試試題

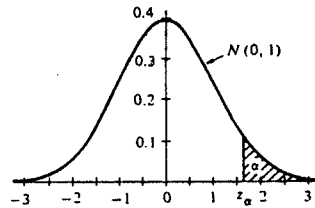
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科目：統計學

Appendix Tables

683

TABLE Vb
The Normal Distribution



z_α	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.4960	0.4920	0.4880	0.4840	0.4801	0.4761	0.4721	0.4681	0.4641
0.1	0.4602	0.4562	0.4522	0.4483	0.4443	0.4404	0.4364	0.4325	0.4286	0.4247
0.2	0.4207	0.4168	0.4129	0.4090	0.4052	0.4013	0.3974	0.3936	0.3897	0.3859
0.3	0.3821	0.3783	0.3745	0.3707	0.3669	0.3632	0.3594	0.3557	0.3520	0.3483
0.4	0.3446	0.3409	0.3372	0.3336	0.3300	0.3264	0.3228	0.3192	0.3156	0.3121
0.5	0.3085	0.3050	0.3015	0.2981	0.2946	0.2912	0.2877	0.2843	0.2810	0.2776
0.6	0.2743	0.2709	0.2676	0.2643	0.2611	0.2578	0.2546	0.2514	0.2483	0.2451
0.7	0.2420	0.2389	0.2358	0.2327	0.2296	0.2266	0.2236	0.2206	0.2177	0.2148
0.8	0.2119	0.2090	0.2061	0.2033	0.2005	0.1977	0.1949	0.1922	0.1894	0.1867
0.9	0.1841	0.1814	0.1788	0.1762	0.1736	0.1711	0.1685	0.1660	0.1635	0.1611
1.0	0.1587	0.1562	0.1539	0.1515	0.1492	0.1469	0.1446	0.1423	0.1401	0.1379
1.1	0.1357	0.1335	0.1314	0.1292	0.1271	0.1251	0.1230	0.1210	0.1190	0.1170
1.2	0.1151	0.1131	0.1112	0.1093	0.1075	0.1056	0.1038	0.1020	0.1003	0.0985
1.3	0.0968	0.0951	0.0934	0.0918	0.0901	0.0885	0.0869	0.0853	0.0838	0.0823
1.4	0.0808	0.0793	0.0778	0.0764	0.0749	0.0735	0.0721	0.0708	0.0694	0.0681
1.5	0.0668	0.0655	0.0643	0.0630	0.0618	0.0606	0.0594	0.0582	0.0571	0.0559
1.6	0.0548	0.0537	0.0526	0.0516	0.0505	0.0495	0.0485	0.0475	0.0465	0.0455
1.7	0.0446	0.0436	0.0427	0.0418	0.0409	0.0401	0.0392	0.0384	0.0375	0.0367
1.8	0.0359	0.0351	0.0344	0.0336	0.0329	0.0322	0.0314	0.0307	0.0301	0.0294
1.9	0.0287	0.0281	0.0274	0.0268	0.0262	0.0256	0.0250	0.0244	0.0239	0.0233
2.0	0.0228	0.0222	0.0217	0.0212	0.0207	0.0202	0.0197	0.0192	0.0188	0.0183
2.1	0.0179	0.0174	0.0170	0.0166	0.0162	0.0158	0.0154	0.0150	0.0146	0.0143
2.2	0.0139	0.0136	0.0132	0.0129	0.0125	0.0122	0.0119	0.0116	0.0113	0.0110
2.3	0.0107	0.0104	0.0102	0.0099	0.0096	0.0094	0.0091	0.0089	0.0087	0.0084
2.4	0.0082	0.0080	0.0078	0.0075	0.0073	0.0071	0.0069	0.0068	0.0066	0.0064
2.5	0.0062	0.0060	0.0059	0.0057	0.0055	0.0054	0.0052	0.0051	0.0049	0.0048
2.6	0.0047	0.0045	0.0044	0.0043	0.0041	0.0040	0.0039	0.0038	0.0037	0.0036
2.7	0.0035	0.0034	0.0033	0.0032	0.0031	0.0030	0.0029	0.0028	0.0027	0.0026
2.8	0.0026	0.0025	0.0024	0.0023	0.0023	0.0022	0.0021	0.0021	0.0020	0.0019
2.9	0.0019	0.0018	0.0018	0.0017	0.0016	0.0016	0.0015	0.0015	0.0014	0.0014
3.0	0.0013	0.0013	0.0013	0.0012	0.0012	0.0011	0.0011	0.0011	0.0010	0.0010
3.1	0.0010	0.0009	0.0009	0.0009	0.0008	0.0008	0.0008	0.0008	0.0007	0.0007
3.2	0.0007	0.0007	0.0006	0.0006	0.0006	0.0006	0.0006	0.0005	0.0005	0.0005
3.3	0.0005	0.0005	0.0005	0.0004	0.0004	0.0004	0.0004	0.0004	0.0004	0.0003
3.4	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0002



國立臺灣科技大學

九十一學年度碩士班招生考試試題

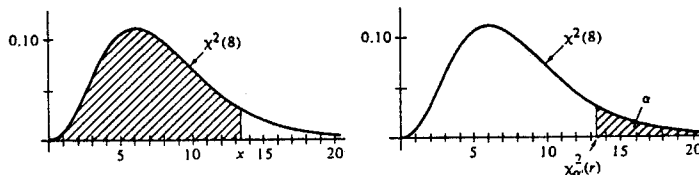
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科目：統計學

Appendix Tables

681

TABLE IV
The Chi-Square Distribution



$$P(X \leq x) = \int_0^x \frac{1}{\Gamma(r/2)2^{r/2}} w^{r/2-1} e^{-w/2} dw$$

r	P(X ≤ x)							
	0.010	0.025	0.050	0.100	0.900	0.950	0.975	0.990
	$\chi^2_{0.99}(r)$	$\chi^2_{0.975}(r)$	$\chi^2_{0.95}(r)$	$\chi^2_{0.90}(r)$	$\chi^2_{0.10}(r)$	$\chi^2_{0.05}(r)$	$\chi^2_{0.025}(r)$	$\chi^2_{0.01}(r)$
1	0.000	0.001	0.004	0.016	2.706	3.841	5.024	6.635
2	0.020	0.051	0.103	0.211	4.605	5.991	7.378	9.210
3	0.115	0.216	0.352	0.584	6.251	7.815	9.348	11.34
4	0.297	0.484	0.711	1.064	7.779	9.488	11.14	13.28
5	0.554	0.831	1.145	1.610	9.236	11.07	12.83	15.09
6	0.872	1.237	1.635	2.204	10.64	12.59	14.45	16.81
7	1.239	1.690	2.167	2.833	12.02	14.07	16.01	18.48
8	1.646	2.180	2.733	3.490	13.36	15.51	17.54	20.09
9	2.088	2.700	3.325	4.168	14.68	16.92	19.02	21.67
10	2.558	3.247	3.940	4.865	15.99	18.31	20.48	23.21
11	3.053	3.816	4.575	5.578	17.28	19.68	21.92	24.72
12	3.571	4.404	5.226	6.304	18.55	21.03	23.34	26.22
13	4.107	5.009	5.892	7.042	19.81	22.36	24.74	27.69
14	4.660	5.629	6.571	7.790	21.06	23.68	26.12	29.14
15	5.229	6.262	7.261	8.547	22.31	25.00	27.49	30.58
16	5.812	6.908	7.962	9.312	23.54	26.30	28.84	32.00
17	6.408	7.564	8.672	10.08	24.77	27.59	30.19	33.41
18	7.015	8.231	9.390	10.86	25.99	28.87	31.53	34.80
19	7.633	8.907	10.12	11.65	27.20	30.14	32.85	36.19
20	8.260	9.591	10.85	12.44	28.41	31.41	34.17	37.57
21	8.897	10.28	11.59	13.24	29.62	32.67	35.48	38.93
22	9.542	10.98	12.34	14.04	30.81	33.92	36.78	40.29
23	10.20	11.69	13.09	14.85	32.01	35.17	38.08	41.64
24	10.86	12.40	13.85	15.66	33.20	36.42	39.36	42.98
25	11.52	13.12	14.61	16.47	34.38	37.65	40.65	44.31
26	12.20	13.84	15.38	17.29	35.56	38.88	41.92	45.64
27	12.88	14.57	16.15	18.11	36.74	40.11	43.19	46.96
28	13.56	15.31	16.93	18.94	37.92	41.34	44.46	48.28
29	14.26	16.05	17.71	19.77	39.09	42.56	45.72	49.59
30	14.95	16.79	18.49	20.60	40.26	43.77	46.98	50.89
40	22.16	24.43	26.51	29.05	51.80	55.76	59.34	63.69
50	29.71	32.36	34.76	37.69	63.17	67.50	71.42	76.15
60	37.48	40.48	43.19	46.46	74.40	79.08	83.30	88.38
70	45.44	48.76	51.74	55.33	85.53	90.53	95.02	100.4
80	53.34	57.15	60.39	64.28	96.58	101.9	106.6	112.3

This table is abridged and adapted from Table III in *Biometrika Tables for Statisticians*, edited by E. S. Pearson and H. O. Hartley. It is published here with the kind permission of the *Biometrika* Trustees.



國立臺灣科技大學

九十一學年度碩士班招生考試試題

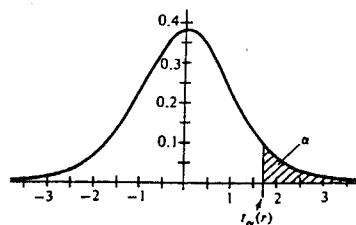
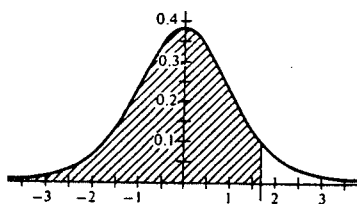
系所組別：工業管理系甲組、乙組、丙組、丁組

科目：統計學

684

Appendix Tables

TABLE VI
The *t* Distribution



$$P(T \leq t) = \int_{-\infty}^t \frac{\Gamma[(r+1)/2]}{\sqrt{\pi r} \Gamma(r/2) (1+w^2/r)^{(r+1)/2}} dw$$

$$[P(T \leq -t) = 1 - P(T \leq t)]$$

<i>r</i>	<i>P</i> (<i>T</i> ≤ <i>t</i>)						
	0.60	0.75	0.90	0.95	0.975	0.99	0.995
	<i>t</i> _{0.40} (<i>r</i>)	<i>t</i> _{0.25} (<i>r</i>)	<i>t</i> _{0.10} (<i>r</i>)	<i>t</i> _{0.05} (<i>r</i>)	<i>t</i> _{0.025} (<i>r</i>)	<i>t</i> _{0.01} (<i>r</i>)	<i>t</i> _{0.005} (<i>r</i>)
1	0.325	1.000	3.078	6.314	12.706	31.821	63.657
2	0.289	0.816	1.886	2.920	4.303	6.965	9.925
3	0.277	0.765	1.638	2.353	3.182	4.541	5.841
4	0.271	0.741	1.533	2.132	2.776	3.747	4.604
5	0.267	0.727	1.476	2.015	2.571	3.365	4.032
6	0.265	0.718	1.440	1.943	2.447	3.143	3.707
7	0.263	0.711	1.415	1.895	2.365	2.998	3.499
8	0.262	0.706	1.397	1.860	2.306	2.896	3.355
9	0.261	0.703	1.383	1.833	2.262	2.821	3.250
10	0.260	0.700	1.372	1.812	2.228	2.764	3.169
11	0.260	0.697	1.363	1.796	2.201	2.718	3.106
12	0.259	0.695	1.356	1.782	2.179	2.681	3.055
13	0.259	0.694	1.350	1.771	2.160	2.650	3.012
14	0.258	0.692	1.345	1.761	2.145	2.624	2.997
15	0.258	0.691	1.341	1.753	2.131	2.602	2.947
16	0.258	0.690	1.337	1.746	2.120	2.583	2.921
17	0.257	0.689	1.333	1.740	2.110	2.567	2.898
18	0.257	0.688	1.330	1.734	2.101	2.552	2.878
19	0.257	0.688	1.328	1.729	2.093	2.539	2.861
20	0.257	0.687	1.325	1.725	2.086	2.528	2.845
21	0.257	0.686	1.323	1.721	2.080	2.518	2.831
22	0.256	0.686	1.321	1.717	2.074	2.508	2.819
23	0.256	0.685	1.319	1.714	2.069	2.500	2.807
24	0.256	0.685	1.318	1.711	2.064	2.492	2.797
25	0.256	0.684	1.316	1.708	2.060	2.485	2.787
26	0.256	0.684	1.315	1.706	2.056	2.479	2.779
27	0.256	0.684	1.314	1.703	2.052	2.473	2.771
28	0.256	0.683	1.313	1.701	2.048	2.467	2.763
29	0.256	0.683	1.311	1.699	2.045	2.462	2.756
30	0.256	0.683	1.310	1.697	2.042	2.457	2.750
∞	0.253	0.674	1.282	1.645	1.960	2.326	2.576

This table is taken from Table III of Fisher and Yates: *Statistical Tables for Biological, Agricultural, and Medical Research*, published by Longman Group Ltd., London (previously published by Oliver and Boyd, Edinburgh), by permission of the authors and publishers.

