國立臺灣科技大學
九十一學年度碩士班招生考試試題

系所組別：工業管理系甲組
科　目：作業研究

總分100分。Show all your calculations.

(1) Consider the following linear programming problem, where the parameter θ must be nonnegative.

$$\begin{aligned} \text{Max} \quad & Z(\theta) = (3+\theta)x_1 + (5-\theta)x_2 + (2+\theta)x_3 \\ \text{s.t.} \quad & x_1 + x_2 \geq 3 + 2\theta \\ & 2x_1 + x_2 + x_3 = 10 - \theta \\ \text{and} \quad & x_1, x_2, x_3 \geq 0. \end{aligned}$$

Let x_4 be the surplus variable for the first functional constraint, and $\bar{x}_5$ and $\bar{x}_6$ be the artificial variables for the respective functional constraints. After we apply the simplex method with the Big-M method and with $\theta = 0$, the final simplex tableau is

Basic var.	Eq. No.	Cofficient							Right Side
		Z	x_1	x_2	x_3	x_4	$\bar{x}_5$	$\bar{x}_6$	
Z	0	1	7	0	3	0	M	5+M	50
x_2	1	0	2	1	1	0	0	1	10
x_4	2	0	1	0	1	1	-1	1	7

Determine the range of nonnegative values of θ over which this basic solution is feasible. (10%) Determine the range of nonnegative values of θ over which this basic solution is both feasible and optimal. (5%) Determine the best choice of θ over this range. (5%)

(2) Consider the following integer nonlinear programming problem. (15%)

$$\begin{aligned} \text{Maximize} \quad & z = x^3 - 2x_1^2 + 2x_2^3 - 3x_2^2 \\ \text{subject to} \quad & x_1 + x_2 \leq 4, \quad \text{and} \\ & x_1, x_2 \geq 0, \\ & x_1 \quad \text{and} \quad x_2 \quad \text{are integers} \end{aligned}$$

Formulate a binary integer programming model where the binary variables have the interpretation,

$$y_{ij} = \begin{cases} 1, & \text{if} \quad x_i = j, \\ 0, & \text{otherwise.} \end{cases}$$

(3) Suppose we have a network with 8 vertices and 13 arcs, where the lengths are specified as follows:

$$\begin{aligned} & d_{12} = 2, \quad d_{13} = 5, \quad d_{14} = 4, \quad d_{23} = 6, \\ & d_{25} = 6, \quad d_{35} = 3, \quad d_{38} = 6, \quad d_{45} = 3, \\ & d_{46} = 4, \quad d_{57} = 3, \quad d_{58} = 2, \quad d_{67} = 2, \\ & d_{78} = 1. \end{aligned}$$

Find the shortest path from vertex 1 to vertex 8 (10%) and the length of the shortest path(5%) (d_{ij} is the distance between vertex i and j).

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(4) At the beginning of a period, a company observes its inventory level, then an order may be placed (and is instantaneously received). Finally, the periods' demand is observed. We are given the following information: (i) A \$2 cost is assessed against each unit of inventory on hand at the end of a period. (ii) A \$3 penalty is assessed against each unit of demand that is not met on time. Assume that all shortages result in lost sales. (iii) Placing an order costs \$0.5 per unit plus a \$5 ordering cost. (iv) During each period, the probability of demand being 1 unit, 2 units and 3 units is 1/3 each. The company is considering the following ordering policy: At the end of any period, if the on-hand inventory is 1 unit or less, order sufficient units to bring the on-hand inventory at the beginning of the next period up to 4 units.
   (a) What fraction of the time will the on-hand inventory level at the end of each period be 0 unit? 1 unit? 2 units? 3 units? 4 units ? (15%)
   (b) Determine the average cost per period incurred by this ordering policy. (15%)

(5) A bank presently has one outside drive-up teller. It takes the teller an average of four minutes to serve a bank customer. Customers arrive at the drive-up window at the rate of 12 per hour. The bank operations officer is currently analyzing the possibility of adding a second drive-up window at an annual cost of \$20,000. It is assumed that arriving cars would be equally divided between both windows. The operations officer estimates that each minute's reduction in customer waiting time would increase the bank's revenue by \$2,000 annually. Should the second drive-up window be installed ? (20%)