國立臺灣科技大學
九十二學年度碩士班招生考試試題
系所組別：自動化及控制研究所碩士班甲組、自動化及控制研究所碩士班丙組
科　目：工程數學

總分 100 分，答案之題號請標示清楚。

1. Determine the eigen values and eigen functions of the Sturm-Liouville Problem: **(15%)**

$x(xy')' + \lambda y = 0$ with boundary conditions of $y(1) = y(e) = 0$.

2. Solve the integral $\int_{-\infty}^{\infty} \frac{\sin^2 \omega}{\omega^2} d\omega$ **(15%)**

(a) using Parseval's theory **(8%)**

$$\int_{-\infty}^{\infty} |f(t)|^2 dt = \frac{1}{2\pi} \int_{-\infty}^{\infty} |\phi(\omega)|^2 d\omega, \quad \text{where} \quad f(t) = \begin{cases} 1 & |t| < 1 \\ 0 & otherwise \end{cases} \quad \text{and} \quad \phi(\omega) = \int_{-\infty}^{\infty} f(t) e^{-j\omega t} dt$$

(b) using Residue theory **(7%)**

3. Solve the initial-value problem **(20%)**

$$\frac{d^2y}{dt^2} + 16\frac{dy}{dt} = 0, \quad y(0) = -\frac{1}{2}, \quad y'(0) = 0.$$

(a) Please transform the equation into matrix form $\mathbf{X}' = \mathbf{AX}$, $\mathbf{X}(0) = \mathbf{K}$. **(5%)**

(b) Please determine e^{At}. **(10%)**

(c) Solve the solution $y(t)$ from (b). **(5%)**

4. Please compare Laplace Transform and Fourier Transform: **(20%)**

(a) Describe the definition of Laplace Transform and Fourier Transform; explain the existence conditions for Laplace Transform and Fourier Transform. **(4%)**

(b) Discuss the transformation relations between Laplace Transform and Fourier Transform? **(6%)**

(c) Given $f(t) = 1 - e^{-10t}u(t)$, determine the Laplace Transform and the Fourier Transform? Discuss the results according to (b) **(10%)**

5. Given complex series $f(z) = \sum_{n=0}^{\infty} \frac{z^n}{2^{n+1}} + \sum_{n=1}^{\infty} \frac{1}{z^n}$ **(15%)**

(a) Is z=0 a singular point? Prove it. **(10%)**

(b) Find the residue at z=0. **(5%)**

6. Evaluate the following problems using Residue theory: **(15%)**

$$\int_0^{\infty} \frac{(\ln x)^2}{a^2 + x^2} dx \quad \text{and} \quad \int_0^{\infty} \frac{\ln x}{a^2 + x^2} dx$$