國立臺灣科技大學
九十二學年度碩士班招生考試試題
系所組別：自動化及控制研究所碩士班乙組
科　目：　統計學

STATISTICS
Apr. 2003

1. (10%) Suppose that a person plays a game in which his score must be one of the 5 numbers $1, 2, \ldots, 5$ and that each of these 5 numbers is equally likely to be his score. The first time he plays the game, his score is X. He then continues to play the game until he obtains another score Y such that $Y \geq X$. Assuming that all plays of the game are independent, Find the probability of the event that $Y = 5$.

2. (20%) A salesman calls prospective buyers to make a sales pitch. Assume that the outcomes of consecutive calls are independent and that on each call he has a 15% chance of making a sale. His daily goal is to make 3 sales, and he can make only 20 calls in a day. To answer the following questions, define the *random variable* X involved and define the *event* in terms of X *or receive no credits*.

   (a) What is the probability that he achieves his goal in 18 trials?

   (b) What is the probability that he does not achieve his daily goal?

3. (20%) Consider a $N(\mu, \sigma^2 = 3^2)$ distribution. To test $H_0 : \mu = 32$ against $H_1 : \mu > 32$, we reject H_0 if the sample mean $\bar{X} \geq c$.

   (a) Find the sample size n and constant c such that $OC(32) = 0.975$ and $OC(\mu = 35) = 0.16$, where $OC(a) = \mathrm{P}(\bar{X} < c | \mu = a)$.

   (b) As n increases, what would happen to constant c and $OC(\mu = 35)$ while keeping $OC(32) = 0.975$?

   ($z_{0.025} \approx 2.0$, $z_{0.16} \approx 1.0$)

4. (10%) The probability p of getting a head when flipping the coin once is estimated by $\hat{p} = \sum_{i=1}^{n} X_i / n$, where

$$X_i = \begin{cases} 1, & \text{if head occurs in the } i\text{th flip} \\ 0, & \text{otherwise.} \end{cases}$$

   Show that $\hat{p}(1 - \hat{p})/(n-1)$ is an unbiased estimator of $\mathrm{Var}(\hat{p})$.

5. (10%) *Random sample* $\{X_1, \ldots, X_n\}$ is drawn from $N(\mu, \sigma^2)$.
   Sample variance $S^2 = \sum_{i=1}^{n} (X_i - \bar{X})^2 / (n-1) = 3$ is observed. Give the probability statement of *p-value* for testing $H_0 : \sigma^2 = 5$ against $H_1 : \sigma^2 \neq 5$.

6. (20%) 報載 1 號候選人的民意調查支持度爲 $\hat{p} = 30\%$, 有效樣本爲 1000 人, 信賴水準爲 95%, 最大抽樣誤差爲 3%。假設另外再抽樣 1000 人之民意調查支持度爲 $\hat{p}$。p 表示投票當天的得票率。判斷下列敍述的眞僞並說明原因。

   (a) $\mathrm{P}(27\% < \hat{p} < 33\%) = 95\%$

   (b) $\mathrm{P}(27\% < p < 33\%) = 95\%$

   (c) p 會介於 27% 與 33%之間

   (d) 兩倍有效樣本可將抽樣誤差降爲一半

7. (10%) 一燈泡工廠所生產之燈泡的壽命具有「常態分布」。根據歷史數據得知燈泡壽命的「標準差」爲 42 小時。由一組 36 個燈泡的「樣本」算得「樣本平均數」爲 780 小時。

   (a) 求算「信賴水準」爲 95% 之平均壽命的「信賴區間」。

   (b) 如果要求「抽樣誤差」小於 12 小時的機率爲 95%, 則需抽樣幾個燈泡?

   ($z_{0.025} \approx 2$)