

國立臺灣科技大學

九十二學年度碩士班招生考試試題

系所組別：機械工程系碩士班甲組、乙組、丙組、丁組、戊組

科目：工程數學

總分 100 分

1. Solve the initial value problem

(a) $y'' + 2y' + 2y = 0; \quad y(0) = 0, y'(0) = 1$ (10)

(b) $y'' + 2y' + 2y = \delta(t-2); \quad y(0) = y'(0) = 0,$ (10)

where $\delta(t)$ is the Dirac-delta function.

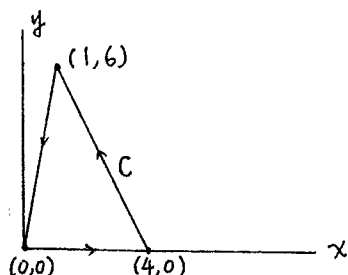
2. (a) Find the eigenvalues and the corresponding eigenvectors of the matrix

$$[A] = \begin{bmatrix} 4 & -2 \\ -2 & 1 \end{bmatrix}. \quad (10)$$

(b) Find an orthogonal matrix $[Q]$ that diagonalizes the matrix $[A]$. That is,determine matrices $[Q]$ and $[D]$ such that $[Q]^{-1}[A][Q] = [D]$ is diagonal.

(10)

3. A particle moves once counterclockwise about the triangle with vertices

 $(0,0), (4,0)$, and $(1,6)$, under the influence of the force $\vec{F} = xy\vec{i} + x\vec{j}$. Calculatethe work done by this force. That is, calculate the integral $\oint_C \vec{F} \cdot d\vec{R}$. (15)4. (a) Find the Fourier sine expansion of $\Phi(x) = 1, 0 \leq x \leq 1$. (10)

(b) Using the results of part (a), find the solution of the boundary value problem

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2} \quad 0 < x < 1$$

 $u(0,t) = 0, u(1,t) = 0$ for $0 < t < \infty$, and

$$u(x,0) = 1 \text{ for } 0 \leq x \leq 1 \quad (15)$$

5. Evaluate the given integral by means of the residue theorem

$$\int_b^{2\pi} \frac{d\theta}{2 - \sin \theta} \quad (20)$$

