

國立臺灣科技大學

九十二學年度碩士班招生考試試題

系所組別：化學工程系碩士班

科目：工程數學

總分 100 分

請標明題號，作答時務必列出過程，否則酌予扣分。

1. A periodical function of the form

$$f(t) = \begin{cases} 1 & 0 < t < 1 \\ 0 & 1 \leq t < 2 \end{cases},$$

and the period of f is 2. Please find the Laplace transform of this function. (15%)2. In a horizontal frictionless mass-spring system, the force exerted is $F = -kx$, where k is a positive spring constant and x is a coordinate representing the displacement of mass m from its equilibrium position.

- (i) By Newton's second law, please write down the differential equation concerning the motion of the mass with respect to time t .
- (ii) Please find the solutions for the differential equation.
- (iii) Actually, the solutions of the differential equation describe "simple harmonic motion" and can also be expressed simply as a sine function. Please show the form of this sine function and its period. (15%)

3. (i) Find the general solution for $\frac{d^2x}{dt^2} + 2\frac{dx}{dt} + 2x = 5\cos t$ (ii) Find the particular solution for the above differential equation if $x = 0$ and

$$\frac{dx}{dt} = 0 \text{ for } t = 0. \quad (15\%)$$

4. Using the method of least squares, fit a straight line to the four points given below: $(-1.3, 0.11)$, $(-0.1, 1.11)$, $(0.2, 0.81)$, $(1.3, 1.90)$. Find the equation of this straight line. (10%)

5. Find the solution of an ordinary differential equation

$$\frac{dx}{dt} = kx\left(1 - \frac{x}{x_\infty}\right) \text{ and } t = 0 \quad x = x_0; \quad x_\infty = \text{const}. \quad (15\%)$$

6. Find a general solution of the nonhomogeneous linear system of differential equations

$$Y' = AY + G = \begin{bmatrix} 2 & -4 \\ 1 & -3 \end{bmatrix} Y + \begin{bmatrix} 2t^2 + 10t \\ t^2 + 9t + 3 \end{bmatrix}, \text{ where } Y' = \begin{bmatrix} y_1' \\ y_2' \end{bmatrix} \quad (15\%)$$

7. By taking Laplace transform to solve the partial differential equation

$$\frac{\partial c}{\partial t} + v \frac{\partial c}{\partial z} = -kc$$

$$c(t, 0) = c_0 \sin \omega t$$

$$c(0, z) = 0$$

(15%)

