國立臺灣科技大學
九十二學年度碩士班招生考試試題
系所組別：資訊工程系碩士班、電子工程系碩士班甲組
科　目：離散數學

總分100分

1. (10%) Please derive the complexity bound, $\log N! > N \log N - N$, using the integration by parts.

2. (a) (5%) Let the two variables A and B be Boolean variables. Is the logical expression, $(A \wedge B) \rightarrow (A \vee B)$, a tautology? (Note that your answer must be shown.) (b) (5%) Please transfer the logical expression, $(A \vee B) \rightarrow (C \wedge D)$, into the disjunction normal form.

3. (10%) Use the generating function technique to solve the second order recurrence $T_n = 5T_{n-1} - 6T_{n-2}$ with the boundary conditions $T_0 = 5$ and $T_1 = 13$. (Note that don't use the characteristic equation technique to solve it.)

4. (10%) Given a Turing machine TM, an input character string '$AAA...A$' have been written on the tape of TM. Please design a Turing Program, i.e. a set of transition functions, to change the whole string '$AAA...A$' to the new string '$BBB...B$'. (Note that before providing your answer, you should define some necessary notations such as the set of states, the halting state, etc.)

5. (10%) Let the cyclic group $(G, *)$ be a finite group with the generator g. Show that it exists k such that $g^k = e$, where e is the unit element in that group.

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6. (a) (6%) A manufacturing process produces a mix of "good" memory chips and "bad" memory chips. The lifetime of good chips follows the exponential law with a rate of failure α. The lifetime of bad chips also follows the exponential law, but the rate of failure is 1000α. Suppose that the fraction of good chips is $1-p$ and of bad chips, p. Find the probability that a randomly selected chip is still functioning after t seconds.

   (b) (10%) Consider the memory chips in (a). In order to "weed out" the bad chips, every chip is tested for t seconds prior to leaving the factory. The chips that fail are discarded and the remaining chips are sent out to customers. Find the value of t for which 99% of the chips sent out to customers are good.

7. (10%) A real number is called rational if it can be written as the quotient of two integers. Show that the set of rational numbers between 0 and 1 is countable.

8. (8%) Show that an inverse of a modulo m does not exist if $\gcd(a, m) > 1$.

9. (8%) Show that the symmetric closure of the union of two relations is the union of their symmetric closures.

10. (8%) Determine whether the given pair of graphs is isomorphic. Prove that your answer is correct.