

國立臺灣科技大學

九十二學年度碩士班招生考試試題

系所組別：資訊工程系碩士班、電子工程系碩士班甲組、電機工程系碩士班丙一組
 科目：資料結構

總分 100 分

18%1. The number of vertices in a graph G is called its order, denoted as p , and the number of edges is its size, denoted as q . A rooted tree is called a binary tree if every vertex has at most 2 children. The root r is located at level 0; the largest level of a vertex in a rooted tree is called its height, denoted as h . Answer the following questions.

- 3%(a). What is the size of a tree of order p ?
- 3%(b). What is the maximum size of a graph G of order p ?
- 3%(c). What is the largest height of a binary tree of order p ?
- 3%(d). What is the smallest height of a binary tree of order p ?
- 3%(e). What is the worst time complexity for searching a keyword in a tree constructed in the form of question 1.(c)?
- 3%(f). What is the worst time complexity for searching a keyword in a tree constructed in the form of question 1.(d)? Note that the key of the left (right) child of any vertex v in a tree is less (larger) than that of the key of vertex v .

12%2. The number of vertices adjacent to vertex v is called the degree of vertex v . A vertex with odd degree is called an odd vertex; otherwise it is called an even vertex. For a graph G with $V(G) = \{v_1, v_2, \dots, v_p\}$, we associate a degree sequence $\deg v_1, \deg v_2, \dots, \deg v_p$, where $\deg v_1 \geq \deg v_2 \geq \dots \geq \deg v_p$. Also we call a sequence of nonnegative integers graphical if it is the degree sequence of some graph. Answer the following questions.

- 3%(a). What is the total degree of a graph G with order p and size q ?
- 3%(b). Does every graph contain an even number of odd vertices?
- 3%(c). What is the degree sequence of Fig. 1?
- 3%(d). 5, 5, 3, 2, 1, 0 is a sequence of six nonnegative integers. Is it graphical?



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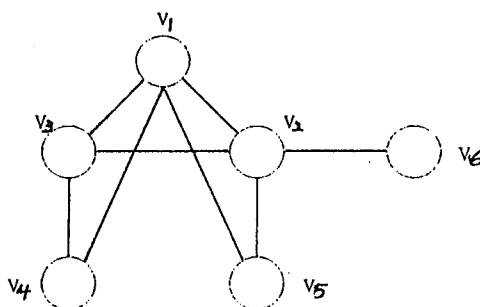


Figure 1.

2083. Assume that a given graph G with $V(G) = \{v_1, v_2, \dots, v_p\}$ is

represented by its adjacency lists, where the adjacency list of a given vertex that the vertices adjacent to that vertex are listed in increasing order of their subscripts. In a depth-first-search of G , the vertex that is currently visited is designed the active vertex and labeled with a depth-first search index $dfi(v)$ to represent the visiting ordering. After the depth-first search algorithm applied for a graph G , there is a depth-first search tree T created. The edge in both G and T is called a tree edge; otherwise it is called a back edge. Answer the following questions.

- 5%(a). What is the relationship of $dfi(u)$ and $dfi(v)$ of a back edge (u, v) ?
- 5%(b). What is the relationship of $dfi(u)$ and $dfi(v)$ of a tree edge (u, v) ?
- 5%(c). The lowpoint $l(v)$ of v is the smallest value $dfi(u)$ of a vertex u of T that can be reached from v by a directed $v \rightarrow u$ path consisting of edges of T followed by at most one back edge. Let T be a depth-first search tree of a connected graph G , and let u be a vertex that is not a root of T . Suppose u is a cut-vertex of G and it has a child v . What is the relationship between $l(v)$ and $dfi(u)$?
- 5%(d). Based on 3.(C), put $l(v)$ for each vertex v in Fig. 2



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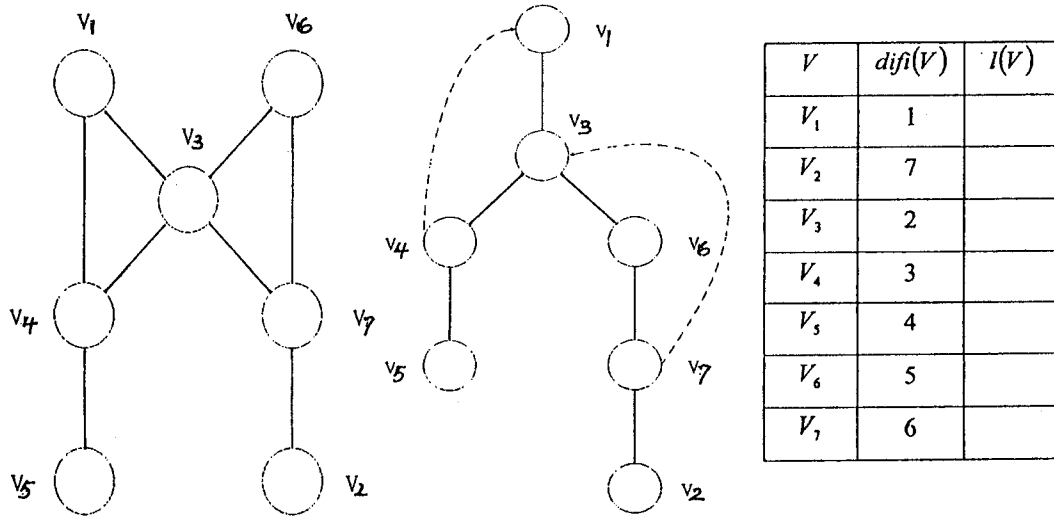


Figure 2

✱請先將此表格畫在答案卷上，再填入答案。

18%4. Show in detail how heapsort processes the input 142, 543, 123, 65, 453, 879, 572, 434, 111, 242, 811, 102.

16%5.

8%(a). Which function grows faster: $N \log N$ or $N^{1+\epsilon/\sqrt{\log N}}$, $\epsilon > 0$? (You need to justify your reason)

8%(b). Solve the following recurrence in Big-Oh notation:

$$T(N) = (1/N) \left[\sum_{i=0}^{N-1} T(i) \right] + cN, T(0) = 0.$$

16%6 Write compact programs/functions to solve Fibonacci sequence using recursive method and iterative method, respectively in C language.

