國立臺灣科技大學
九十二學年度碩士班招生考試試題
系所組別：電子工程系碩士班乙一組
科　目：　通信系統

總分共 100 分，作答時請於答案卷上標明題號

1. Let $x(t)$ denote a baseband signal with bandwidth W Hz, and let $X(f)$ denote the Fourier transform of $x(t)$. Suppose $p(t)$ denote a periodic pulse signal of duty-cycle Δ as defined by the formula

$$p(t) = \sum_{k=-\infty}^{\infty} [u(t - T\Delta - kT) - u(t - kT)]$$

where $u(t)$ is the unit step function.

(a). (6%) Please compute the Fourier transform of $x(t)\,p(t)$.

(b). (6%) We intend to build a DSB-SC amplitude modulation system by feeding the signal $x(t)p(t)$ through a suitable bandpass filter. Suppose $T = \frac{1}{5W}$ and $\Delta = 0.15$. It is required that the peak-to-peak amplitude of the modulation system output is identical to that of the corresponding baseband signal. Please plot the magnitude response of the desired bandpass filter for a given feasible DSB-SC channel. In the plot, please clearly indicate the corner frequencies and the gain at the passband, the transition band, and the stopband.

(c). (4%) Please explain the main difficulties in realizing the above mentioned DSB-SC system.

2. Suppose continuous-time signals $x(t)$ and $y(t)$ are known to have bandwidths B_1 Hz and B_2 Hz, respectively.

(a). (6%) Please state the sampling theorem and prove it through the use of the Fourier transform of the sampled continuous-time signal.

(b). (6%) Please verify that the constraint on the sampling interval T_0 for the formula

$$y(t) = \int_{-\infty}^{\infty} x(\tau)y(t-\tau)\,d\tau = T_0 \sum_{k=-\infty}^{\infty} x(kT_0)\,y(t-kT_0)$$

to hold true for all t is that $T_0(B_1 + B_2) < 1$.

(c). (6%) Suppose we intend to reconstruct the signal $z(t) = x(t)y(t)$ from sampled signals $x(nT_1)$ and $y(nT_2)$. It is desired to reconstruct the signal $z(t)$ without any aliasing error, but at a minimum combined data rate (samples/sec). Please determine the sampling intervals T_1 and T_2, and plot a block diagram for the signal reconstruction system.

3. Let $s(t)$ be a wide-sense stationary Gaussian random process with zero mean and the autocorrelation function defined as

$$R(\tau) = \begin{cases} 1 - \frac{|\tau|}{T}, & |\tau| < T \\ 0, & |\tau| \geq T. \end{cases}$$

It is known that the random process $s(t)$ is ergodic. Suppose that Θ is a uniform random variable distributed on $[0,\ 2\pi)$ and that A is a uniform random variable distributed on $[0,\ 1]$. Furthermore, it is assumed that Θ, A and $s(t)$ are independent. Let us define a random signal $x(t)$ as

$$x(t) = A\,s(t)\,\sin(2\pi f_0 t + \Theta).$$

(a). (5%) Please compute the autocorrelation function of $x(t)$.

國立臺灣科技大學
九十二學年度碩士班招生考試試題
系所組別：電子工程系碩士班乙一組
科　目：　通信系統

(b). (5%) Please determine the power spectral density function of $x(t)$.

(c). (3%) Determine whether $x(t)$ is ergodic and verify your answer.

(d). (3%) Determine whether $x(t)$ is wide-sense stationary and verify your answer.

4. A (5,2) linear block code (referred to as $\mathcal{C}$) is generated by $\underline{c} = \underline{m}\,\underline{\underline{G}}$, where $\underline{m}$ is the message word, $\underline{c}$ is the codeword and the generator matrix $\underline{\underline{G}}$ is given by

$$\underline{\underline{G}} = \begin{bmatrix} 1 & 1 & 0 & 1 & 1 \\ 1 & 0 & 1 & 0 & 1 \end{bmatrix}.$$

(a). (3%) List all codewords of $\mathcal{C}$.

(b). (3%) What is the minimum distance of $\mathcal{C}$?

(c). (3%) In $\mathcal{C}$, up to how many erroneous bits in one codeword can be corrected ?

(d). (4%) If [01111] is received, which codeword and message word should it be decoded into ?

(e). (3%) Is $\mathcal{C}$ a cyclic code ?

(f). (3%) Is $\mathcal{C}$ a systematic code ?

5. Let us consider 8-PSK modulation and demodulation.

(a). (5%) How many bits of information are carried in one transmitted symbol ?

(b). (5%) In order to reduce the bit error probability (also called bit error rate (BER)) as much as possible under a power constraint, the data bits are Gray-coded into transmitted symbols. Then, what is the symbol error probability that corresponds to a BER of 10^{-4} ?

(c). (5%) A receiver structure for the 8-PSK demodulation is to be constructed with as few correlators as possible. How many correlators do we need ?

6. In BPSK (binary phase shift keying) signal transmission, logical 0 and logical 1 are, respectively, mapped into waveforms $s_0(t) = A\cos 2\pi f_c t$ and $s_1(t) = -A\cos 2\pi f_c t$ for $0 < t < T$. Assume that the communication is through an AWGN (additive white Gaussian noise) channel, where the two-sided power spectral density of the noise is $N_0/2$.

(a). (5%) What is the null-to-null bandwidth, which is equal to the width of the main lobe of the signal power spectrum, consumed by this BPSK transmission ? Please express your answer in terms of T.

(b). (5%) The bandwidth efficiency is defined to be the ratio between the data rate (i.e. number of bits transmitted in one second) and the consumed bandwidth (measured in Hz= 1/sec). Let us adopt the null-to-null bandwidth here. What is the bandwidth efficiency of BPSK modulation ?

(c). (6%) For $A = 3$, $T = 1$ and $N_0 = 1/2$, find the BER of the BPSK modulation. Please express your answer in terms of the Q function, which is defined by $Q(z) = \text{Prob}(Z > z)$, where Z is the standard Gaussian random variable.