國立臺灣科技大學
九十二學年度碩士班招生考試試題
系所組別：電子工程系碩士班乙二組
科　目：線性系統

總分爲 100 分，答題時請標各大題號以及小題號

1. Justify your answer or give a counterexample in this problem.
   (a) Both x(t) and y(t) are periodic then x(t) + y(t) is always periodic.(2%)
   (b) A system being two nonlinear systems cascaded is always nonlinear. (2%)
   (c) Only LTI system has impulse response. (2%)
   (d) The Fourier series coefficients of a periodic, continuous-time signal can not be periodic. (2%)
   (e) Let the Laplace transform of two signals x(t) and y(t) be X(s) and Y(s) with region of convergence (ROC) R_1 and R_2, respectively. Then the Laplace transform of x(t) + y(t) is with ROC being $R_1 \cap R_2$.(2%)
2. An LTI system is causal if and only if the unit step response S[n] is zero for n < 0. Prove it or give a counterexample. (5%)
3. Prove your answer: (a) Consider a time-invariant, nonlinear system with input x(t) and output y(t), if x(t) is periodic, then y(t) is also periodic too? (5%) (b) An LTI system, if the input is not periodic, then output is not periodic either? (5%)
4. If a system has input x(t) and output y(t) with the relation y(t) = [cos(3t)]x(t), determine whether the system is (a) memoryless(2%), (b) time-invariant(2%), (c) linear(2%), (d) causal(2%), (e) stable(2%), (f) the period of x(t) is $\frac{\pi}{2}$, is y(t) periodic? If yes, find the period (5%).
5. Let x[n] and y[n] be two real-valued discrete-time signals. The autocorrelation function $\phi_{xx}[n]$ and the cross-correlation $\phi_{xy}[n]$ are given as $\phi_{xx}[n] = \sum_{m=-\infty}^{\infty} x[m+n]x[m]$ and $\phi_{xy}[n] = \sum_{m=-\infty}^{\infty} x[m+n]y[m]$. Let x[n] be the input to the LTI system with impulse response h[n] and the corresponding output be y[n], find the expressions of $\phi_{xy}[n]$ and $\phi_{yy}[n]$ in terms of $\phi_{xx}[n]$ and h[n].(10%)
6. Let h(t) be the impulse response of a causal and stable LTI system with a rational system function. H(s) is the Laplace transform of h(t). (a) Is the system with impulse response $dh(t)/dt$ guaranteed to be causal and stable? (5%) (b) Is the system with impulse response $\int_{-\infty}^{t} h(\tau)d\tau$ guaranteed to be causal and unstable? (5%)
7. Let x[n] denote a causal sequence (i.e. x[n] = 0, n < 0) for which x[0] is nonzero and finite and X(z) is the z-transform of x[n]. (a) Is that possible X(z) having poles or zeros at z =∞ (5%) (b) Find x[1] in terms of X(z) and x[0]. (5%)
8. Let x(t) be a band-limited signal with Nyquist sampling rate ω_0. (a) what is the maximum frequency of x(t). (2%) Determine the Nyquist rate each of the following signals:

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(b) x(t) + x(t - 1). (2%) (c) $dx(t)/dt$ (2%). (d) $x^2(t)$. (2%). (e) $x(t)\cos\omega_0 t$ (2%).

9. Given the Fourier transform of u(t) is $\pi\delta(\omega)+1/j\omega$. Let x(t) be a signal whose Fourier transform is $X(\omega)=\delta(\omega)+\delta(\omega-\pi)+\delta(\omega-5)$, and let h(t) = u(t) – u(t - 2). (a) Find the Fourier transform of h(t)(3%). (b) Is x(t) periodic?(3%) (c) Find y(t) = x(t)*h(t). "*" denotes convolution. (3%) (d) Can the convolution of two aperiodic signals be periodic? (3%)

10. Find the Fourier transforms in frequency domain f, **not in** ω **domain,** of sgn(t) given below as $\text{sgn}(t)=\begin{cases}1 & t>0\\ -1 & t<0\end{cases}$ (2%)

11. (a). Find the discrete-time Fourier of $(1/2)^n u[n]$. (2%) (b) Find $\sum_{n=0}^{\infty} n(1/2)^n$.(4%)