

國立臺灣科技大學

九十二學年度碩士班招生考試試題

系所組別：電機工程系碩士班乙一組

科目：工程數學

(共九題；滿分一百分)

1. Consider a differential equation as $\frac{dP}{dt} = P(t)(c_1 - c_2P(t))$, where c_1 and c_2 are constants. Find the solution for the differential equation given $P(0) = P_0$. (10 points)
2. If both $\mu_1(x, y) = xy$ and $\mu_2(x, y) = (x^2 + y^2)^{-1}$ are integrating factors for the differential equation $y' = f(x, y)$, then what is $f(x, y)$? (10 points)
3. Let $\Phi(x)$ and $\Psi(x)$ be linearly independent solutions of $y'' + p(x)y' + q(x)y = 0$ on an open interval I . Assume that $p(x)$ and $q(x)$ are continuous on I . Then prove that between two consecutive zeros of $\Phi(x)$, there always exists exact one zero for $\Psi(x)$. (15 points)
4. Solve $-t(1+t)y'' + 2y' + 2y = 6(t+1)$; $y(-1) = y(1) = 0$. (15 points)



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5. Describe all solutions of $Ax = 0$ in a parametric vector form, where

A is the following matrix. (10%)

$$A = \begin{bmatrix} 1 & -5 & 0 & 2 & 0 & -4 \\ 0 & 0 & 1 & 0 & 0 & -3 \\ 0 & 0 & 0 & 0 & 4 & 8 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

6. Find the inverse matrix of the following matrix, if it exists. (10%)

$$\begin{bmatrix} 1 & 3 & 2 \\ 0 & 1 & 1 \\ -1 & 0 & 1 \end{bmatrix}$$

7. Given a matrix with its row equivalent matrix shown below, decide bases for $\text{Col } A$ and $\text{Nul } A$. (10%)

$$A = \begin{bmatrix} 1 & -3 & 2 & 5 \\ -2 & 6 & 0 & -3 \\ 4 & -12 & -4 & -1 \end{bmatrix} \sim \begin{bmatrix} 1 & -3 & 2 & 5 \\ 0 & 0 & 4 & 7 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

8. Let $A = \{a_1, a_2, a_3\}$ and $B = \{b_1, b_2, b_3\}$ be bases for the vector space V , and suppose that $a_1 = 4b_1 - b_2$, $a_2 = -b_1 + b_2 + b_3$, and $a_3 = b_2 - 2b_3$.

(a) Find the change-of-coordinate matrix from A to B . (5%)

(b) Find $[x]_B$ for $x = 3a_1 + 4a_2 + a_3$. (5%)

9. Define $T: \mathbb{R}^2 \rightarrow \mathbb{R}^2$ by $T(x) = Ax$, where $A = \begin{bmatrix} 7 & 2 \\ -4 & 1 \end{bmatrix}$. Find a

base B for \mathbb{R}^2 with the property that the B -matrix of T is a diagonal matrix. (10%)

