

國立臺灣科技大學

九十二學年度碩士班招生考試試題

系所組別：電機工程系碩士班丙一組、電機工程系碩士班丙二組

科目：線性代數與機率

總分 100 分

1. Answer the following questions. (10%)

(a) Are the vectors  $a = \begin{pmatrix} 2 \\ 1 \\ 0 \end{pmatrix}$ ,  $b = \begin{pmatrix} 3 \\ 2 \\ 1 \end{pmatrix}$ ,  $c = \begin{pmatrix} 5 \\ 3 \\ 1 \end{pmatrix}$  linearly independent? Explain why.

(b)  $\begin{pmatrix} a & b & 0 \\ c & d & 0 \\ 0 & 0 & 1 \end{pmatrix}^{-1} = ?$       (c)  $A = \begin{pmatrix} 11 & 0 & 0 & 3 \\ 12 & 2 & 10 & 4 \\ 13 & 0 & 1 & 5 \\ 4 & 0 & 0 & 0 \end{pmatrix}$ ,  $\det(A) = ?$

2. Find the quadratic polynomial  $p(t) = a + bt + ct^2$  which best fits the data: (10%)

$t$	-2	-1	0	1	2
$p(t)$	-4	-1	0	0	0

3. Consider the 2 by 2 matrix,  $C = \begin{pmatrix} 27 & -20 \\ 34 & -25 \end{pmatrix}$ . (10%)

(a) find the eigenvalues of C.

(b) for each eigenvalue of C, find an eigenvector of C corresponding to this eigenvalue.

4. For the ellipse  $6x_1^2 + 4x_1x_2 + 3x_2^2 = 1$ , find: (10%)

(a) The principal axes;

(b) The equation of the ellipse in the coordinate system given by the principal axes;

(c) The lengths of the semi axes.



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5. Find an orthogonal basis of the subspace  $W$  of  $R^5$  orthogonal to  $w_1 = \langle 1, 1, 3, 4, 1 \rangle$ ,  $w_2 = \langle 1, 2, 1, 2, 1 \rangle$ ,  $w_3 = \langle 1, 1, 1, 2, 1 \rangle$ . (10%)
6. A box contains 4,000 components, of which 5% are defective. A second box contains 800 components of which 40% are defective. Two other boxes contain 1,500 components each, with 15% defective components. We select at random one of the above boxes and from it at random a single component. What is the probability that this component is defective. (15%)
7. Suppose Taiwan has 3,000,000 automobiles registered. The license plates are numbered serially, beginning with 1 (not really, but let's assume it for now). As one gentleman crossed Keelung Road on Friday afternoon, a car run him down (too bad!). What is the probability that the first digit on the license plate of the car will be the digit 1? Please state any assumptions made. (20%)
8. Four cards containing the numbers 1, 2, 3, and 123 respectively, are shuffled and one card is drawn. We define the following events:
- a) The card drawn contains the number 1.
  - b) The card drawn contains the number 2.
  - c) The card drawn contains the number 3
- Are these three events independent? Prove your answer. (15%)

