國立臺灣科技大學

九十二學年度碩士班招生考試試題

系所組別：資訊工程系碩士班、電子工程系碩士班甲組、電機工程系碩士班丙一組

科　目：資料結構

總分100分

18%1. The number of vertices in a graph G is called its order, denoted as p, and the number of edges is its size, denoted as q. A rooted tree is called a binary tree if every vertex has at most 2 children. The root r is located at level 0; the largest level of a vertex in a rooted tree is called its height, denoted as h. Answer the following questions.

3%(a). What is the size of a tree of order p?

3%(b). What is the maximum size of a graph G of order p?

3%(c). What is the largest height of a binary tree of order p?

3%(d). What is the smallest height of a binary tree of order p?

3%(e). What is the worst time complexity for searching a keyword in a tree constructed in the form of question 1.(c)?

3%(f). What is the worst time complexity for searching a keyword in a tree constructed in the form of question 1.(d)? Note that the key of the left (right) child of any vertex v in a tree is less (larger) than that of the key of vertex v.

12%2. The number of vertices adjacent to vertex v is called the degree of vertex v. A vertex with odd degree is called an odd vertex; otherwise it is called an even vertex. For a graph G with $V(G)=\{v_1, v_2, \ldots, v_p\}$, we associate a degree sequence deg v_1, deg v_2, ..., deg v_p, where deg $v_1 \geq$ deg $v_2 \geq \ldots \geq$ deg v_p. Also we call a sequence of nonnegative integers graphical if it is the degree sequence of some graph. Answer the following questions.

3%(a). What is the total degree of a graph G with order p and size q?

3%(b). Does every graph contain an even number of odd vertices?

3%(c). What is the degree sequence of Fig. 1?

3%(d). 5, 5, 3, 2, 1, 0 is a sequence of six nonnegative integers. Is it graphical?

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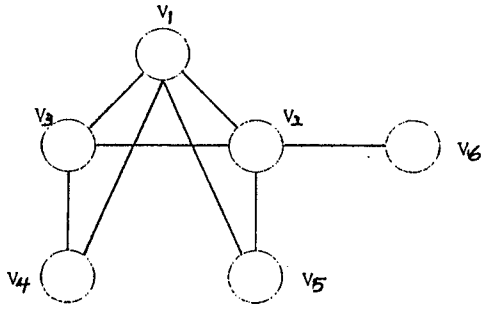

Figure 1.

20%3. Assume that a given graph G with $V(G)=\{v_1, v_2, \ldots, v_p\}$ is represented by its adjacency lists, where the adjacency list of a given vertex that the vertices adjacent to that vertex are listed in increasing order of their subscripts. In a depth-first-search of G, the vertex that is currently visited is designed the active vertex and labeled with a depth-first search index $dfi(v)$ to represent the visiting ordering. After the depth-first search algorithm applied for a graph G, there is a dephth-first search tree T created. The edge in both G and T is called a tree edge; otherwise it is called a back edge. Answer the following questions.

5%(a). What is the relationship of $dfi(u)$ and $dfi(v)$ of a back edge (u,v)?

5%(b). What is the relationship of $dfi(u)$ and $dfi(v)$ of a tree edge (u,v)?

5%(c). The lowpoint $l(v)$ of v is the smallest value $dfi(u)$ of a vertex u of T that can be reached from v by a directed $v \rightarrow u$ path consisting of edges of T followed by at most one back edge. Let T be a depth-first search tree of a connected graph G, and let u be a vertex that is not a root of T. Suppose u is a cut-vetex of G and it has a child v. What is the relationship between $l(v)$ and $dfi(u)$?

5%(d). Based on 3.(C), put $l(v)$ for each vertex v in Fig. 2

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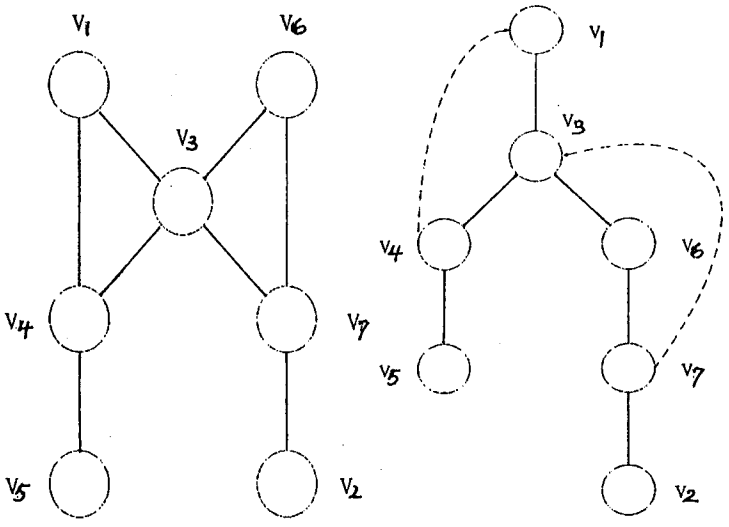
※請先將此表格畫在答案卷上，再填入答案。



V	$difi(V)$	$l(V)$
V_1	1	
V_2	7	
V_3	2	
V_4	3	
V_5	4	
V_6	5	
V_7	6	

Figure 2

18%4. Show in detail how heapsort processes the input 142, 543, 123, 65, 453, 879, 572, 434, 111, 242, 811, 102.

16%5.

8%(a). Which function grows faster: $N\log N$ or $N^{1+\varepsilon/\sqrt{\log N}}, \varepsilon > 0$? (You need to justify your reason)

8%(b). Solve the following recurrence in Big-Oh notation:

$$T(N) = (1/N)[\sum_{i=0}^{N-1} T(i)] + cN, T(0) = 0 .$$

16%6. Write compact programs/functions to solve **Fibonacci** sequence using *recursive* method and *iterative* method, respectively in *C* language.