

國立臺灣科技大學

九十二學年度碩士班招生考試試題

系所組別：資訊工程系碩士班、電子工程系碩士班甲組

科目：離散數學

總分 100 分

1. (10%) Please derive the complexity bound,  $\log N! > N \log N - N$ , using the integration by parts.
2. (a) (5%) Let the two variables  $A$  and  $B$  be Boolean variables. Is the logical expression,  $(A \wedge B) \rightarrow (A \vee B)$ , a tautology? (Note that your answer must be shown.) (b) (5%) Please transfer the logical expression,  $(A \vee B) \rightarrow (C \wedge D)$ , into the disjunction normal form.
3. (10%) Use the generating function technique to solve the second order recurrence  $T_n = 5T_{n-1} - 6T_{n-2}$  with the boundary conditions  $T_0 = 5$  and  $T_1 = 13$ . (Note that don't use the characteristic equation technique to solve it.)
4. (10%) Given a Turing machine  $TM$ , an input character string 'AAA...A' have been written on the tape of  $TM$ . Please design a Turing Program, i.e. a set of transition functions, to change the whole string 'AAA...A' to the new string 'BBB...B'. (Note that before providing your answer, you should define some necessary notations such as the set of states, the halting state, etc.)
5. (10%) Let the cyclic group  $(G, *)$  be a finite group with the generator  $g$ . Show that it exists  $k$  such that  $g^k = e$ , where  $e$  is the unit element in that group.



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6. (a) (6%) A manufacturing process produces a mix of “good” memory chips and “bad” memory chips. The lifetime of good chips follows the exponential law with a rate of failure  $\alpha$ . The lifetime of bad chips also follows the exponential law, but the rate of failure is  $1000\alpha$ . Suppose that the fraction of good chips is  $1-p$  and of bad chips,  $p$ . Find the probability that a randomly selected chip is still functioning after  $t$  seconds.
- (b) (10%) Consider the memory chips in (a). In order to “weed out” the bad chips, every chip is tested for  $t$  seconds prior to leaving the factory. The chips that fail are discarded and the remaining chips are sent out to customers. Find the value of  $t$  for which 99% of the chips sent out to customers are good.
7. (10%) A real number is called rational if it can be written as the quotient of two integers. Show that the set of rational numbers between 0 and 1 is countable.
8. (8%) Show that an inverse of  $a$  modulo  $m$  does not exist if  $\gcd(a, m) > 1$ .
9. (8%) Show that the symmetric closure of the union of two relations is the union of their symmetric closures.
10. (8%) Determine whether the given pair of graphs is isomorphic. Prove that your answer is correct.

