

國立臺灣科技大學

九十二學年度碩士班招生考試試題

系所組別：資訊管理系碩士班甲組

科目：離散數學

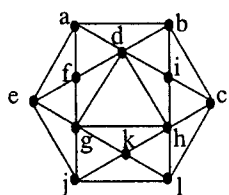
題目共 8 題，總分 100 分

1. (15 %)

$$\text{Let } F_n = \frac{1}{\sqrt{5}} \left[\left(\frac{1+\sqrt{5}}{2} \right)^n - \left(\frac{1-\sqrt{5}}{2} \right)^n \right]$$

Use mathematical induction to prove that F_n is always an integer for every non-negative integer n .

2. (15%) Consider an undirected graph as the following:



- Find an Euler cycle from this graph, or prove there is no Euler cycle in this graph. (7%)
- Find a Hamiltonian cycle from this graph, or prove there is no Hamiltonian cycle in this graph. (8%)

3. (10 %) Prove that:

$$\sum_{k=0}^{\infty} kx^k = \frac{x}{(1-x)^2} \text{ for } |x| < 1$$

- (10 %) Define $\phi(n)$ to be the number of positive integers less than n which are relatively prime to n . Find the value of $\phi(16200)$. (Hint: you may make use of the theorem that $\phi(mn) = \phi(m)\phi(n)$ if m, n are relatively prime)



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5. (15%) Give the following truth table of a circuit (T for True and F for False):

Case	p	q	r	Result
1	T	T	T	T
2	T	T	F	T
3	T	F	T	F
4	T	F	F	F
5	F	T	T	T
6	F	T	F	F
7	F	F	T	F
8	F	F	F	T

- (a). Use the disjunctive normal form to express the circuit. (5%)
 (b). Use the conjunctive normal form to express the circuit. (5%)
 (c). Simplify the circuit using Karnaugh map. (5%)
6. (10%) Let the set $A = \{1, 2, 3, 4, 5\}$ and let the relation R on $A \times A$ be defined as $R = \{(1, 1), (2, 2), (1, 2), (2, 1), (2, 4), (3, 5), (4, 2), (5, 3)\}$.
 (a). Is R is reflective? Why? (5% , 本小題全對才給分)
 (b). Is R is antisymmetric? Why? (5% , 本小題全對才給分)
7. (10%) Let A be the set of integers, B be the set of real numbers, and $f: A \rightarrow B$ be defined by $f(x) = 3x + 2$. Give only YES or NO for answering the following statements: (2% for each)
 (a). f is injective.
 (b). f is surjective.
 (c). f is bijective.
 (d). A is countable.
 (e). B is countable.
8. (15%) Solve the following recursive function:
 $a_0 = 2$, $a_1 = 6$, $a_n = 4a_{n-1} - 4a_{n-2}$ for $n > 2$.

