

國立臺灣科技大學

九十二學年度碩士班招生考試試題

系所組別：資訊管理系碩士班乙組

科目：統計學

題目共 9 題，總分 100 分

- (10%) Suppose that a person plays a game in which his score must be one of the 5 numbers  $1, 2, \dots, 5$  and that each of these 5 numbers is equally likely to be his score. The first time he plays the game, his score is  $X$ . He then continues to play the game until he obtains another score  $Y$  such that  $Y \geq X$ . Assuming that all plays of the game are independent, Find the probability of the event that  $Y = 5$ .
- (10%) Consider a  $N(\mu, \sigma^2 = 3^2)$  distribution. To test  $H_0: \mu = 32$  against  $H_1: \mu > 32$ , we reject  $H_0$  if the sample mean  $\bar{X} \geq c$ .
  - Find the sample size  $n$  and constant  $c$  such that  $OC(32) = 0.975$  and  $OC(\mu = 35) = 0.16$ , where  $OC(a) = P(\bar{X} < c | \mu = a)$ . (5%)
  - As  $n$  increases, what would happen to constant  $c$  and  $OC(\mu = 35)$  while keeping  $OC(32) = 0.975$ ? (5%)

( $z_{0.025} \approx 2.0$ ,  $z_{0.16} \approx 1.0$ )
- (10%) The probability  $p$  of getting a head when flipping the coin once is estimated by  $\hat{p} = \sum_{i=1}^n X_i/n$ , where

$$X_i = \begin{cases} 1, & \text{if head occurs in the } i\text{th flip} \\ 0, & \text{otherwise.} \end{cases}$$

Show that  $\hat{p}(1 - \hat{p})/(n - 1)$  is an unbiased estimator of  $\text{Var}(\hat{p})$ .

- (10%) Random sample  $\{X_1, \dots, X_n\}$  is drawn from  $N(\mu, \sigma^2)$ . Sample variance  $S^2 = \sum_{i=1}^n (X_i - \bar{X})^2 / (n - 1) = 3$  is observed. Give the probability statement of  $p$ -value for testing  $H_0: \sigma^2 = 5$  against  $H_1: \sigma^2 \neq 5$ .
- (10%) 報載 1 號候選人的民意調查支持度為  $\hat{p} = 30\%$ ，有效樣本為 1000 人，信賴水準為 95%，最大抽樣誤差為 3%。假設另外再抽樣 1000 人之民意調查支持度為  $\hat{p}$ ， $p$  表示投票當天的得票率。判斷下列敘述的真偽並說明原因。
  - (3%)  $P(27\% < \hat{p} < 33\%) = 95\%$
  - (3%)  $P(27\% < p < 33\%) = 95\%$
  - (2%)  $p$  會介於 27% 與 33% 之間
  - (2%) 兩倍有效樣本可將抽樣誤差降為一半



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6. The probability density function of  $X$  is given by

$$f(x) = \begin{cases} 2e^{2(1-x)}, & x > 1 \\ 0, & x \leq 1. \end{cases}$$

- (a) (7%) Find the *moment generating function* (m.g.f.) of  $X$ .  
 (b) (4%+4%) Find the *mean* and *variance* of  $X$  by utilizing the m.g.f. obtained in (a).  
 7. (10%) Let  $X$  be the number of heads occurred when flipping a coin with  $P(\text{head}) = p$   $N$  times. Suppose that  $N$  has a Poisson distribution with mean  $\lambda$ . Find the distribution of  $X$ .  
 8. An experiment is conducted to investigate the relationship between the *electrical conductivity* (EC) of the soil and the yield of tomatoes (kg/plot). Below are the summary data:

|             | EC level |      |      |
|-------------|----------|------|------|
|             | 1.6      | 3.8  | 6.0  |
| $n_i$       | 4        | 4    | 4    |
| $\bar{Y}_i$ | 59.5     | 55.4 | 50.7 |
| $s_i^2$     | 6.7      | 7.1  | 6.3  |

- (a) (8%) Construct the appropriate *analysis of variance* table.  
 (b) (4%) Test the hypothesis that there are no effects due to the electrical conductivity of the soil. Use  $\alpha = 0.05$ .  
 (c) (3%) Calculate the  $R$ -square value and explain it.  
 9. A box is known to contain 6 balls. Those balls are either red or white. The number of red balls  $\theta$  is unknown. Three balls are drawn without replacement and 1 red ball is found. Answer the following questions.  
 (a) (8%) Calculate the probabilities of obtaining this result.  
 (b) (2%) From the answer of (a), show that the *maximum likelihood estimate* for  $\theta$  is 2.

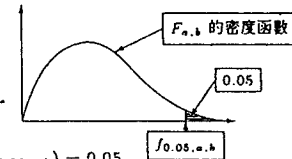


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表 A.5(a) 「自由度」為  $(a, b)$  之  $F$ -分配的 95 百分位數  $f_{0.05, a, b}$ ,  $P(F_{a, b} > f_{0.05, a, b}) = 0.05$ 

| b   | a      |        |        |        |        |        |        |        |        |        |
|-----|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|
|     | 1      | 2      | 3      | 4      | 5      | 6      | 7      | 8      | 9      | 10     |
| 1   | 161.45 | 199.50 | 215.71 | 224.58 | 230.16 | 233.99 | 236.77 | 238.88 | 240.54 | 241.88 |
| 2   | 18.51  | 19.00  | 19.16  | 19.25  | 19.30  | 19.33  | 19.35  | 19.37  | 19.38  | 19.40  |
| 3   | 10.13  | 9.55   | 9.28   | 9.12   | 9.01   | 8.94   | 8.89   | 8.85   | 8.81   | 8.79   |
| 4   | 7.71   | 6.94   | 6.59   | 6.39   | 6.26   | 6.16   | 6.09   | 6.04   | 6.00   | 5.96   |
| 5   | 6.61   | 5.79   | 5.41   | 5.19   | 5.05   | 4.95   | 4.88   | 4.82   | 4.77   | 4.74   |
| 6   | 5.99   | 5.14   | 4.76   | 4.53   | 4.39   | 4.28   | 4.21   | 4.15   | 4.10   | 4.06   |
| 7   | 5.59   | 4.74   | 4.35   | 4.12   | 3.97   | 3.87   | 3.79   | 3.73   | 3.68   | 3.64   |
| 8   | 5.32   | 4.46   | 4.07   | 3.84   | 3.69   | 3.58   | 3.50   | 3.44   | 3.39   | 3.35   |
| 9   | 5.12   | 4.26   | 3.86   | 3.63   | 3.48   | 3.37   | 3.29   | 3.23   | 3.18   | 3.14   |
| 10  | 4.96   | 4.10   | 3.71   | 3.48   | 3.33   | 3.22   | 3.14   | 3.07   | 3.02   | 2.98   |
| 11  | 4.84   | 3.98   | 3.59   | 3.36   | 3.20   | 3.09   | 3.01   | 2.95   | 2.90   | 2.85   |
| 12  | 4.75   | 3.89   | 3.49   | 3.26   | 3.11   | 3.00   | 2.91   | 2.85   | 2.80   | 2.75   |
| 13  | 4.67   | 3.81   | 3.41   | 3.18   | 3.03   | 2.92   | 2.83   | 2.77   | 2.71   | 2.67   |
| 14  | 4.60   | 3.74   | 3.34   | 3.11   | 2.96   | 2.85   | 2.76   | 2.70   | 2.65   | 2.60   |
| 15  | 4.54   | 3.68   | 3.29   | 3.06   | 2.90   | 2.79   | 2.71   | 2.64   | 2.59   | 2.54   |
| 16  | 4.49   | 3.63   | 3.24   | 3.01   | 2.85   | 2.74   | 2.66   | 2.59   | 2.54   | 2.49   |
| 17  | 4.45   | 3.59   | 3.20   | 2.96   | 2.81   | 2.70   | 2.61   | 2.55   | 2.49   | 2.45   |
| 18  | 4.41   | 3.55   | 3.16   | 2.93   | 2.77   | 2.66   | 2.58   | 2.51   | 2.46   | 2.41   |
| 19  | 4.38   | 3.52   | 3.13   | 2.90   | 2.74   | 2.63   | 2.54   | 2.48   | 2.42   | 2.38   |
| 20  | 4.35   | 3.49   | 3.10   | 2.87   | 2.71   | 2.60   | 2.51   | 2.45   | 2.39   | 2.35   |
| 21  | 4.32   | 3.47   | 3.07   | 2.84   | 2.68   | 2.57   | 2.49   | 2.42   | 2.37   | 2.32   |
| 22  | 4.30   | 3.44   | 3.05   | 2.82   | 2.66   | 2.55   | 2.46   | 2.40   | 2.34   | 2.30   |
| 23  | 4.28   | 3.42   | 3.03   | 2.80   | 2.64   | 2.53   | 2.44   | 2.37   | 2.32   | 2.27   |
| 24  | 4.26   | 3.40   | 3.01   | 2.78   | 2.62   | 2.51   | 2.42   | 2.36   | 2.30   | 2.25   |
| 25  | 4.24   | 3.39   | 2.99   | 2.76   | 2.60   | 2.49   | 2.40   | 2.34   | 2.28   | 2.24   |
| 26  | 4.23   | 3.37   | 2.98   | 2.74   | 2.59   | 2.47   | 2.39   | 2.32   | 2.27   | 2.22   |
| 27  | 4.21   | 3.35   | 2.96   | 2.73   | 2.57   | 2.46   | 2.37   | 2.31   | 2.25   | 2.20   |
| 28  | 4.20   | 3.34   | 2.95   | 2.71   | 2.56   | 2.45   | 2.36   | 2.29   | 2.24   | 2.19   |
| 29  | 4.18   | 3.33   | 2.93   | 2.70   | 2.55   | 2.43   | 2.35   | 2.28   | 2.22   | 2.18   |
| 30  | 4.17   | 3.32   | 2.92   | 2.69   | 2.53   | 2.42   | 2.33   | 2.27   | 2.21   | 2.16   |
| 40  | 4.08   | 3.23   | 2.84   | 2.61   | 2.45   | 2.34   | 2.25   | 2.18   | 2.12   | 2.08   |
| 60  | 4.00   | 3.15   | 2.76   | 2.53   | 2.37   | 2.25   | 2.17   | 2.10   | 2.04   | 1.99   |
| 120 | 3.92   | 3.07   | 2.68   | 2.45   | 2.29   | 2.18   | 2.09   | 2.02   | 1.96   | 1.91   |
| ∞   | 3.84   | 3.00   | 2.60   | 2.37   | 2.21   | 2.10   | 2.01   | 1.94   | 1.88   | 1.83   |

