

國立臺灣科技大學  
九十三學年度碩士班考試試題

系所組別：自動化及控制研究所甲組、自動化及控制研究所丙組  
科目：工程數學

總分為 100 分

1. Solve the following differential equations:

$$(1) \quad xy' = \frac{y^2}{x} + y \quad (10\%)$$

$$(2) \quad x^2 y'' - 2xy' + 2y = \ln(x) + 1 \quad (10\%)$$

2. Use Laplace Transform method to solve the following equations.

$$x' + y' - x = \cos 2t$$

$$x' + 2y' = 0$$

$$\text{where } x(0) = 0, \quad y(0) = 0 \quad (15\%)$$

3. Consider a matrix  $A = \begin{bmatrix} 2 & 2 \\ 3 & 1 \end{bmatrix}$

(1) Find the eigenvalues. (4%)

(2) Diagonalize the matrix A. (6%)

(3) Compute  $A^8 + A$ . (5%)

4. (a) Find the potential function of the vector field in cylindrical coordinates,

$$\vec{F} = 2(rz - \cos\theta)\vec{e}_r + 2\sin\theta\vec{e}_\theta + r^2\vec{e}_z. \quad (10\%)$$

(b) Calculate the integral  $\iiint_S \frac{x}{4} \vec{R} \cdot \vec{N} dS$  in terms of the volume V and the  $x$ -coordinate of its centroid, where V is the volume bounded by a regular, closed surface S with an outer normal  $\vec{N}$ , and  $\vec{R}$  is a position vector. (10%)

5. Solve the Dirichlet problem in polar coordinates

$$\nabla^2 \phi = 0, \quad 1 \leq r \leq 2 \quad \text{and} \quad 0 \leq \theta \leq \frac{\pi}{3} \quad (15\%)$$

with the boundary conditions  $\phi(r, 0) = \phi(r, \frac{\pi}{3}) = \phi(1, \theta) = 0,$

$$\phi(2, \theta) = 6 \sin \theta - 8 \sin^3 \theta.$$

6. Evaluate the integral  $\int_0^\infty \frac{\sin^2 x}{x^2} dx$  by the residue theorem. (15%)

