

國立臺灣科技大學
九十三學年度碩士班考試試題

系所組別：高分子工程系丙組
科 目：工程數學

總分 100 分

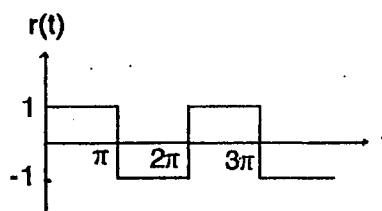
1. Solve the following differential equation: (15 %)

Forced oscillation

$$y'' + y' + 10y = r(t)$$

$$r(t) = \begin{cases} 1 & (0 < t < \pi) \\ -1 & (\pi < t < 2\pi) \end{cases}$$

$$r(t + 2\pi) = r(t)$$



2. Solve the following differential equation: (15 %)

$$x \sin \theta \, d\theta + (x^2 - 2s^2 \cos \theta + \cos \theta) dx = 0$$

3. Use the Laplace transform to solve the system of differential equations. (17%)

$$x_1'' + 10x_1 - 4x_2 = 0, \quad x_1(0) = 0, x_1'(0) = 1$$

$$x_2'' + 4x_1 + 4x_2 = 0, \quad x_2(0) = 0, x_2'(0) = -1$$

4. Transform the conic section $2x^2 + 4xy - y^2 = 1$ to principal axes. (18%)

5. Find the orthogonal trajectories to the given one parameter family of curves (10%)

$$y = cx^2 e^{2x} \text{ with } c \text{ a parameter}$$

6. Consider the boundary-value problem introduced in the construction of the mathematical model for the shape of a rotating string: (13%)

$$\frac{d}{dx} \left[T(x) \frac{dy}{dx} \right] + \rho \omega^2 y = 0 \quad y(1) = 0, \quad y(e) = 0$$

where $T(x) = x^2$ and ρ is a constant. Define the critical speeds of angular rotation ω_h as the values of ω for which the boundary-value problem has nontrivial solutions. For $\rho \omega^2 > 0.25$, find the critical speeds ω_h and the corresponding deflection curves $y_h(x)$.

7. Use the change of variables $y(x) = x^{1/2} u(x^2/2)$ to obtain the general solution of the equation

$$y'' + x^2 y = 0$$

in terms of Bessel functions.

(12%)

