

國立臺灣科技大學
九十三學年度碩士班考試試題

系所組別：化學工程系
科 目：工程數學

總分 100 分，請詳列計算過程並依序作答。

1. If $\mathbf{F}(x, y, z) = f(x, y, z)\mathbf{i} + g(x, y, z)\mathbf{j} + h(x, y, z)\mathbf{k}$
is a continuous vector field whose components have continuous first and second partial derivatives.

Show that $\nabla \cdot (\nabla \times \mathbf{F}) = 0$ (10%)

2. Find the solution of a differential equation
 $y'' + y' = 1 + \delta(t-2); y(0) = 0; y'(0) = 3$ (15%)

3. Find the solution of a differential equation
 $x^2 y'' + xy' + y = \sec(\ln x)$ (15%)

4. Let D be the region bounded by the cylinder $x^2 + y^2 = 16$ and the planes $z = 1$ and $z = 5$.

Find the outward flux $\iint_S (\mathbf{F} \cdot \mathbf{n}) dS$ of the given vector field \mathbf{F} across the close surface S .

$\mathbf{F} = y^2\mathbf{i} + xz^3\mathbf{j} + (z-1)^2\mathbf{k}$ (10%)

5. Find one power series solution of the 2nd order ordinary differential equation $x \frac{d^2 y}{dx^2} + \frac{dy}{dx} + y = 0$, write down the 2nd independent solution in function form without actually determine the recurrence formula. (15%)

6. Find 3 orthonormal eigen vectors of the symmetric matrix $A = \begin{bmatrix} 2 & 2 & 1 \\ 2 & 5 & 2 \\ 1 & 2 & 2 \end{bmatrix}$ (20%)

7. Find the inverse Laplace transform of $\frac{1}{s \cosh(as)}$ where $\cosh(x) = \frac{e^x + e^{-x}}{2}$ [Hint: $\cosh(z) = \cos(iz)$

where $i = \sqrt{-1}$] (15%)

