國立臺灣科技大學
九十三學年度碩士班考試試題

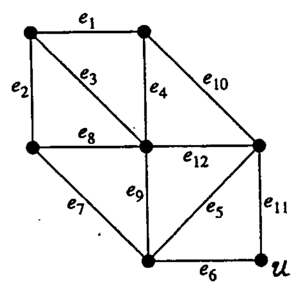
系所組別：電子工程系甲組
科　　目：離散數學

總分 100 分，各題配分標於題後.

1. (a) Let $g(n)=\dfrac{6n^3+1}{2n+1}$. Prove that $g(n)$ is $O(n^2)$ but not $O(n)$.

   (b) Let $s(n)=3^{n+5}$. Find a number a with $s(n)=\Theta(a^n)$. (10%)

2. Consider the following weighted graph, where $w(e_1) < w(e_2) < \cdots < w(e_{11}) < w(e_{12})$.



   (a) Use Kruskal's algorithm to obtain the minimal spanning tree. List the edges in the order in which Kruskal's algorithm will select them.

   (b) Use Prim's algorithm, starting at vertex u, to obtain the minimal spanning tree. List the edges in the order in which Prim's algorithm will select them. (10%)

3. The complement of a graph G is the graph with vertex set $V(G)$ and with an edge between distinct vertices v and w if G does not have an edge joining v and w.

   (a) Show that if G is not connected, then its complement is connected.

   (b) Is the converse to the statement in part (a) true? Show your answer.

   (c) Give an example of a graph that is isomorphic to its complement. (10%)

4. Let f be a function from X to Y. Prove that f is one - to - one if and only if $f(A\cap B)=f(A)\cap f(B)$ for all subsets A and B of X. (10%)

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5. (a) Let p, q, r be propositions. Use **logical equivalence rules** to prove that $(q \to p) \wedge (r \to p)$ is logically equivalent to $(q \vee r) \to p$. DO NOT USE TRUTH TABLES.
   (b) Let Z be the set of all integers. Write the negation of the following statement $p(x, y)$: $\forall x\ \exists y\ [(x \leq y) \wedge (x + y = 0)]$ for all $x, y \in Z$.
   (c) For the statement $p(x, y)$ in (b), determine which is true — $p(x, y)$ or its negation? Why?
   (15%)

6. Let S be the set of all sequences of 0's, 1's, and 2's of length 10. For example, S contains 0211012201.
   (a) How many sequences in S have exactly three 0's?
   (b) How many sequences in S have at least one 0, at least one 1, and at least one 2? (15%)

7. Given positive integer p, define relation R_p on Z as $R_p = \{(m,n): m \equiv n \pmod p\}$.
   (a) Show that R_6 is an equivalence relation, and find the corresponding equivalence classes.
   (b) Find the smallest equivalence relation containing $R_6 \cup R_8$. Justify your answer.
   (c) Let $[Z]_p$ denote the set of equivalence classes corresponding to R_p. Does the formula
   $f([m]_6) = m^2$ give a well-defined function $f : [Z]_6 \to Z$? Justify your answer.
   (15%)

8. Let A be the smallest set of binary strings such that:
   - $10 \in A$, and
   - if $a \in A$, then $a00 \in A$.

   Let $eval : A \to N$ be the function that maps each binary string, a, to the number that has a as a binary representation.

   Prove inductively that, for all $a \in A$, $\log_2(eval(a))$ is odd. (15%)