

國立臺灣科技大學  
九十三學年度碩士班考試試題

系所組別：電子工程系乙二組  
科 目：線性系統

(總分為 100 分 作答請標題號)

1. Consider a system with input  $x[n]$  and output  $y[n]$ . The input/output relation is given by the following two properties (i)  $y[n] - ay[n-1] = x[n]$ , where  $a \neq 0$ , (ii)  $y[0] = 1$ . (a) Find the impulse response  $h[n]$ . (5%) (b) Is the system time-invariant or not? Justify your answer (5%) (c) Is the system linear or not? Justify your answer (5%). (d) If  $y[0] = 0$ , is the system linear or not? Justify your answer (5%)
2. (a) Find  $\delta(at)$  in terms of  $\delta(t)$  if  $a \neq 0$ . (5%). (b) Is that possible an aperiodic input get a periodic output from a system? Justify your answer (5%)
3. (a) Let  $y(t) = e^{-t}u(t) * \sum_{k=-\infty}^{\infty} \delta(t-3k)$ , \* denotes convolution. Find  $y(t)$  for  $0 \leq t < 3$ . (5%) (b) If  $y[n] = x[n] * h[n]$ , then  $y[n-1] = x[n-1] * h[n-1]$ . Is this true? Justify your answer (5%) (c) If  $x(t) = 0$  for  $|t| > T_1$  and  $h(t) = 0$  for  $|t| > T_2$ . Find the range of  $t$  such that  $x(t) * h(t) = 0$ . (5%) (d) If  $x(t) = e^{-5t}u(t)$  to a continuous-time, linear, and time-invariant system resulting in output  $y(t) = \sin \omega_0 t$ , find the impulse response. (5%)
4. Suppose we are given the following information about a signal  $x(t)$ , (1)  $x(t)$  is a real signal, (2)  $x(t)$  is periodic with  $T = 6$  and has Fourier coefficient  $a_k$ , (3)  $a_k = 0$  for  $k=0$  and  $k > 2$ , (4)  $x(t) = -x(t-3)$ , (5)  $\frac{1}{6} \int_3^9 |x(t)|^2 dt = \frac{1}{2}$ , (6)  $a_1$  is a positive number. Use the above conditions to determine  $x(t)$ . (10%)
5. (a) Can the Fourier series coefficient  $a_k$  of a periodic signal be periodic too? Justify your answer (5%) (b) A signal whose Fourier transform  $X(\omega)$  is real, even, and positive. If the maximum value of  $X(\omega)$  is at  $\omega=0$ , i.e.  $X(0)$ , then  $|x(t)| \leq a$ . Find  $a$ . (5%) (c) The moments  $m_n$  of a function  $f(t)$  are defined by  $m_n = \int_{-\infty}^{\infty} t^n f(t) dt$ . The mean  $\eta$  and variance  $\sigma^2$  are given by  $\eta = \frac{m_1}{m_0}$  and  $\sigma^2 = \frac{m_2}{m_0} - \eta^2$ . If the Fourier transform of  $f(t)$  is  $F(\omega)$ , find  $\eta$  and  $\sigma^2$  in terms of  $F(\omega)$ . (5%)
6. Determine whether each of the following statements is true or false. If it is true, construct a convincing argument for it. If it is false, give a counterexample. (a) The Laplace transform of  $t^2 u(t)$  does not converge anywhere on the s-plane. (3%) (b) The Laplace transform of  $e^{t^2} u(t)$  does not converge anywhere on the s-plane. (3%) (c) The Laplace transform of  $e^{j\omega t}$  does not converge anywhere on the s-plane. (3%) (d) The Laplace transform of  $e^{j\omega t} u(t)$  does not converge anywhere on the s-plane. (3%) (e) The Laplace transform of  $|t|$  does not converge anywhere on the s-plane. (3%)
7. (a) What is Gibbs phenomenon? (5%) (b) State and prove Nyquist sampling rate. (5%)

