

國立臺灣科技大學
九十三學年度碩士班招生考試試題

系所組別：電機工程系乙一組

科目：工程數學

總分 100 分

1. (15%) Solve the following systems

$$x'' - 2x' + 3y' + 2y = 4$$

$$2y' - x' + 3y = 0$$

$$x(0) = x'(0) = y(0) = 0$$

2. (15%) Find the general solution of

$$y'' - 3y' + 2y = 2x + 8\sin(2x)$$

3. (10%) For the following equation, write out the first six nonzero terms of a series solution about 0.

$$y'' - 2y' + x^3y = 0$$

4. (10%) Solve the following equation

$$y' = -\frac{1}{x}y^2 + \frac{2}{x}y; \quad y(1) = 4$$



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5. (10%) Let $T: \mathbb{R}^2 \rightarrow \mathbb{R}^3$ be a linear transformation such that

$$T(x_1, x_2) = (x_1 + x_2, -x_1 - 3x_2, -3x_1 - 2x_2)$$

Find $x \in \mathbb{R}^2$ such that $T(x) = (-4, 7, 0)$.

6. (10% with 5% each) Let $T: \mathbb{R}^2 \rightarrow \mathbb{R}^2$ be the transformation that rotates each point in \mathbb{R}^2 about the origin through an angle φ , with counterclockwise rotation for a positive angle.

(a) Find the standard matrix A of this rotation.

(b) Express the matrix $\begin{pmatrix} a & -b \\ b & a \end{pmatrix}$, where a and b are both real numbers, in terms of a rotation transformation.

7. (10%) The set $B = \{1+t^2, t+t^2, 1+2t+t^2\}$ is a basis for the vector space P_2 of polynomials up to the second order. Find the coordinate vector of $P(t) = 1+4t+7t^2$ relative to B .

8. (20%, with 10% each.) Find the invertible matrix P and matrix C of the form

$\begin{pmatrix} a & -b \\ b & a \end{pmatrix}$ for the matrix

$$A = \begin{pmatrix} 1 & 5 \\ -2 & 3 \end{pmatrix}$$

such that the given matrix has the form of $A = PCP^{-1}$.

- (a) What is the matrix P ?
(b) What is the matrix C ?

