

國立臺灣科技大學
九十三學年度碩士班考試試題

系所組別：電機工程系丙一組、電機工程系丙二組
科 目：線性代數與機率

總分 100 分

1. (24%)

(a) (6%) Let $\{v_1, \dots, v_n\}$ be linearly independent vectors in a vector space V prove

that $\{v_2, \dots, v_n\}$ cannot span vector space V .

(b) (6%) Prove that if A and B are invertible and similar, then A^{-1}, B^{-1} are similar.

(c) (6%) If the eigenvalues and the eigenvectors of matrix A are λ_i and ϕ_i ($i=1, \dots, N$) respectively, find the eigenvalues and the eigenvectors of matrix A^n .

(d) (6%) If $T: R^3 \rightarrow R^3$ is a linear operator, and if the kernel of T is a line through the origin, then what kind of geometric object is the range of T ? Explain your reasoning.

2. (12%) A linear system is given by

$$\begin{aligned} x_1 - x_2 &= 4 \\ 3x_1 + 2x_2 &= 1 \\ -2x_1 + 4x_2 &= 3 \end{aligned}$$

(a) (6%) Find the least square solution of the linear system.

(b) (6%) Find the orthogonal projection of \mathbf{b} on the column space of A .

3. (14%) A differential equation system $\begin{cases} \dot{y}_1 = 4y_1 + y_3 \\ \dot{y}_2 = -2y_1 + y_2 \\ \dot{y}_3 = -2y_1 + y_3 \end{cases}$

(a) (8%) Solve the system.

(b) (6%) Find the solution with the initial conditions $y_1(0) = -1, y_2(0) = 1, y_3(0) = 0$.



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4. (10%) A binary (0 or 1) message is transmitted through a communication channel. Due to the noise in the channel, a 0 can be received as a 1 with probability 0.2, and a 1 can be received as a 0 with probability 0.1.
- (a) (5%) Suppose that the source transmits a 0 with probability 0.6 and transmits a 1 with probability 0.4. A 0 is observed. What is the probability that a 0 was transmitted?
- (b) (5%) Suppose that errors in different symbol transmissions are independent and the string of four symbols 0010 is transmitted, what is the probability that at least one symbol is not received correctly?
5. (20%) Two independent, continuous random variables, X and Y , are each uniformly distributed between 1 and 2.
- (a) (10%) Let $Z = 3X^2 + 1$. Find the probability density function for Z .
- (b) (10%) Let $W = X - 2Y$. Find the mean and variance of W .
6. (20%) Consider a sequence of independent flips of a coin for which the probability of heads is P and the probability of tails is $1 - P$.
- (a) (10%) Compute the probability that there is at least one uninterrupted run of four heads if a coin is flipped seven times.
- (b) (10%) Let K be the number of heads that occur on the first N flips of the coin. Find the mean and variance of K .

