

國立臺灣科技大學
九十三學年度碩士班考試試題

系所組別：工業管理系甲組、工業管理系乙組、工業管理系丙組、工業管理系丁組
科目：統計學

共 6 題，總分 100 分。

(15 %)

1. (1) State the Markov's inequality clearly. (5 %)

(2) Prove that the inequality

$$\Pr(X \geq 1, Y \geq 1) \leq \min\{E(X), E(Y)\}$$

holds for any two non-negative continuous random variables X and Y with joint density $f(x, y)$, where X is not necessarily independent of Y and $\min\{a, b\}$ equals the smaller value between a and b . (10%)

(15 %)

2. Let X and Y have the joint density

$$f(x, y) = c, \quad x \geq 0, \quad y \geq 0, \quad 0 \leq x \leq cy \leq 2c.$$

(1) Find the appropriate value of c . (5 %)

(2) Find the marginal distribution of X . (5 %)

(3) Compute the value of $\Pr(0.5 < Y < 1.5 | X = 1)$. (5 %)

(20 %)

3. Let X_1, X_2, \dots, X_n be a random sample from uniform distribution with density

$$f(x) = \frac{1}{\theta}, \quad 0 < x < \theta, \quad \theta > 0.$$

(1) Find the maximum likelihood estimator $\hat{\theta}$ of θ . (5 %)

(2) Compute the cumulative distribution function (c.d.f.) of $\hat{\theta}$. (5 %)

(3) Use $\hat{\theta}$ to construct the 95 % equal-tail confidence interval for θ . (10 %)



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4. A certain change in a manufacturing procedure for component parts is being considered. Samples are taken using both the existing and the new procedure in order to determine if the new procedure results in an improvement. If 75 of 1500 items from the existing procedure were found to be defective and 80 of 2000 items from the new procedure were found to be defective, find a 90% confidence interval for the true difference in the fraction of defectives between the existing and the new process. (15)
5. A manufacturer of sports equipment has developed a new synthetic fishing line that he claims has a mean breaking strength of 8 kilograms with a standard deviation of 0.5 kilogram. Test the hypothesis that $\mu = 8$ kilograms against the alternative that $\mu \neq 8$ kilograms if a random sample of 50 lines is tested and found to have a mean breaking strength of 7.8 kilograms. Use a 0.01 level of significance. (15)
6. In a shop study, a set of data was collected to determine whether or not the proportion of defectives produced by workers was the same for the day, evening, or night shift worked. The following data were collected:

	Shift		
	Day	Evening	Night
Defectives	45	55	70
Nondefectives	905	890	870

Use a 0.025 level of significance to determine if the proportion of defectives is the same for all three shifts. (20)



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Entry is area A under the standard normal curve from -∞ to z(A)

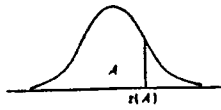


Table of cumulative probabilities for the standard normal distribution. Columns represent z values from .00 to .09. Rows represent cumulative probability values from .0000 to .9999.

Selected Percentiles table. Columns: Cumulative probability A (.90, .95, .975, .98, .99, .995, .999). Rows: z(A) values (1.282, 1.645, 1.960, 2.054, 2.326, 2.576, 3.090).

Entry is $\chi^2(A; \nu)$ where $P(\chi^2(\nu) \leq \chi^2(A; \nu)) = A$

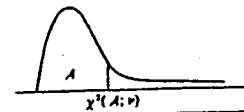


Table of chi-square distribution percentiles. Columns represent probability A values (.005, .010, .025, .050, .100, .900, .950, .975, .990, .995). Rows represent degrees of freedom nu values from 1 to 100.

Source: Reprinted, with permission, from C. M. Thompson, "Table of Percentage Points of the Chi-Square Distribution," Biometrika 32 (1941), pp. 188-89.

Entry is t(A; \nu) where P(t(\nu) \leq t(A; \nu)) = A

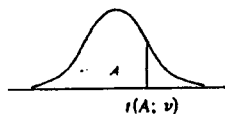


Table of t-distribution percentiles. Columns represent probability A values (.60, .70, .80, .85, .90, .95, .975, .99, .985, .99, .9925, .995, .9975, .9995). Rows represent degrees of freedom nu values from 1 to 60.

