

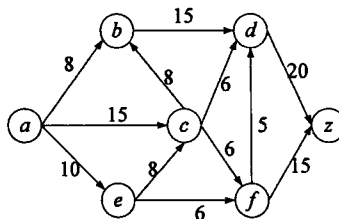
國立臺灣科技大學
九十三學年度碩士班考試試題

系所組別：資訊管理系甲組
科 目：離散數學

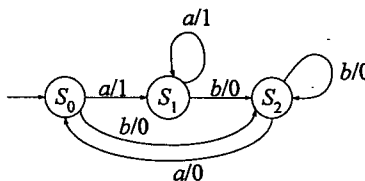
(本試卷共五大題，第一大題含 10 題單選題，第二、三、四、五題為計算題，滿分共計 100 分) ※ 所有答案務必寫於答案卷上，寫於試題上不予計分

一、單選題 (50 points. For each question, +5 if correct, -1 if wrong, and 0 if blank)

- (1) What is the value of $1^2 + 3^2 + 5^2 + \cdots + 25^2$? ① 2925 ② 2926 ③ 2927 ④ 2928 ⑤ 2929
- (2) What is the range for the number of solutions to the inequality $x_1 + x_2 + \cdots + x_4 < 8, x_i \geq 0, 1 \leq i \leq 4$. ① [0,100) ② [100,200) ③ [200,300) ④ [300,400) ⑤ [400,500)
- (3) Given 9 Perl books, 15 Visual BASIC books, 6 Java books, 12 SQL books, and 18 C++ books, how many of these books must we select to insure that we have 10 books dealing with the same computer language? ① 40 ② 43 ③ 46 ④ 49 ⑤ 52
- (4) A connected, planar graph has ten vertices having degrees 2, 2, 2, 3, 3, 3, 4, 4, 4, 5. How many faces are there? ① 4 ② 5 ③ 6 ④ 7 ⑤ 8
- (5) What is the conjunctive normal form of the Boolean function $x \vee \overline{xy}z$?
① $(x \vee y \vee z)(x \vee \overline{y} \vee z)(x \vee y \vee \overline{z})$ ② $(\overline{x} \vee y \vee z)(x \vee \overline{y} \vee z)(x \vee y \vee \overline{z})$
③ $(x \vee \overline{y} \vee z)(x \vee y \vee \overline{z})(x \vee \overline{y} \vee \overline{z})$ ④ $(\overline{x} \vee \overline{y} \vee z)(\overline{x} \vee y \vee \overline{z})(x \vee \overline{y} \vee z)$
⑤ $(x \vee y \vee z)(\overline{x} \vee \overline{y} \vee \overline{z})(\overline{x} \vee y \vee z)$
- (6) There are two sets A and B , where $A - B = \{1, 3, 7, 11\}$, $B - A = \{2, 6, 8\}$, and $A \cap B = \{4, 9\}$. How many numbers are there in the set $A \cup B$? ① 3 ② 5 ③ 7 ④ 9 ⑤ 11
- (7) What is the maximum flow from a to z for the below network? ① 29 ② 30 ③ 31 ④ 32 ⑤ 33



- (8) For finite sets A, B with $|A|=2$ and $|B|=2$, how many relations are there from A to B ? ($|X|$ denotes the number of elements in set X) ① 1 ② 2 ③ 4 ④ 8 ⑤ 16
- (9) For the below finite-state machine, let x be an input string with length 3. If 0 is a suffix of the output string, how many possibilities are there for the string x ? ① 4 ② 5 ③ 6 ④ 7 ⑤ 8



- (10) Let G be the group with subgroups H and K . If $|G|=462$, $|K|=33$, and $K \subset H \subset G$, what is the possible value for $|H|$? ($|X|$ denotes the number of elements in group X) ① 66 ② 99 ③ 142 ④ 165 ⑤ 198



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二、(15 points) Evaluate $\sum_{i=0}^5 \binom{10}{i} \binom{5}{5-i}$

三、(15 points) Solve a_n in the following recurrence relation

$$a_n = a_{n-1} a_{n-2}$$

$$a_0 = 1, a_1 = 2$$

四、(10 points) Find the least positive integer satisfying the following set of congruences:

$$x \equiv 4 \pmod{5}$$

$$x \equiv 3 \pmod{4}$$

$$x \equiv 2 \pmod{7}$$

五、(10 points) If $\langle G, * \rangle$ is a group, and $\forall a \in G, a^2 = a * a = e$ (the identity of G), prove:

(a) (5 points) G is a commutative (abelian) group

(b) (5 points) the inverse of $a * b$ is $b * a$

