

國立臺灣科技大學

九十四學年度碩士班招生考試試題

系所組別：自動化及控制研究所碩士班甲組、自動化及控制研究所碩士班丙組

科 目：工程數學

總分 100 分，答案題號請標示清楚。

1. The following equations (15%)

$$\begin{cases} 3x'' + x + y'' + 3y = e^t \\ 2x'' + x + y'' + 2y = e^{-t} \end{cases}$$

has the initial conditions of $x(0) = y(0) = 1$, $x'(0) = y'(0) = 0$.

Please find $x(t)$ and $y(t)$ using Laplace Transformation.

2. Solve the following linear differential equations (15%)

$$\begin{cases} \frac{dx}{dt} = 3x - y \\ \frac{dy}{dt} = 9x - 3y \end{cases}$$

3. (20%)

(a) Determine e^{At} using Cayley-Hamilton theorem if: (10%)

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 2 \\ 0 & 0 & 2 \end{bmatrix}.$$

(a) Please determine P such that $P^{-1}AP$ is diagonal if:(10%)

$$A = \begin{bmatrix} 5 & -4 & 4 \\ 12 & -11 & 12 \\ 4 & -4 & 5 \end{bmatrix}$$



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4. (19%)

(a) Briefly describe the differences among Taylor series, MacLaurin series, and Laurent series. (5%)

(b) Write the Laurent expansion of the function $f(z) = \frac{2}{1+z^2}$ about the point $z=i$ in an annulus $0 < |z-i| < R$. (9%)

(c) Continue from (b), determine the radius of convergence R and the residue of $f(z)$ at $z=i$. (5%)

5. (18%)

Use the Residue theorem to find the answers to the following two integrals

(a) $\oint_{|z|=2\pi} \tan z dz$ (9%)

(b) $\int_0^{\infty} \frac{\cos 3x}{x^2+1} dx$ (9%)

6. (13%)

If $\Phi = \frac{xy}{z}$ and $A = \sin x \cdot \hat{i} + \cos x \cdot \hat{j} + xy \cdot \hat{k}$, verify the following identities

(a) $\text{curl grad } \Phi = 0$ (4%)

(b) $\text{div curl } A = 0$ (4%)

(c) $\text{div}(\Phi A) = (\text{grad } \Phi) \cdot A + \Phi(\text{div } A)$ (5%)

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