

國立臺灣科技大學

九十四學年度碩士班招生考試試題

系所組別：機械工程系碩士班甲組、乙組、丙組、丁組、戊組
 科目：工程數學

總分 100 分

1. Find the general solution of the differential equation

$$y'' + 2y' + y = \frac{1}{x} e^{-x} \quad (20\%)$$

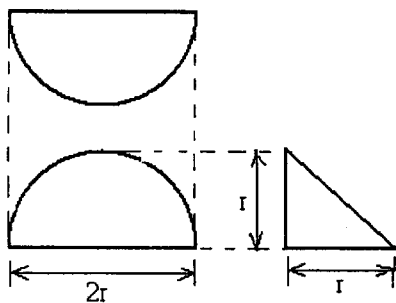
2. Find the general solution of the system

$$\begin{aligned} x_1' &= 3x_1 + 3x_2 + 8, \\ x_2' &= x_1 + 5x_2 + 4e^{3t} \end{aligned} \quad (20\%)$$

3. Consider the vector field
- $\mathbf{F} = 3x^2(y^2 - 4y)\mathbf{i} + (2x^3y - 4x^3)\mathbf{j}$
- .

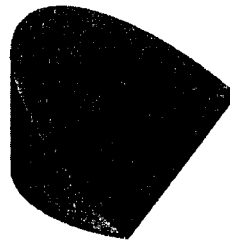
(a) Determine whether \mathbf{F} is conservative in the entire plane. If it is, find a potential function.(b) Evaluate $\int_c \mathbf{F} \cdot d\mathbf{R}$, where c is any path from the first point $(-1, 1)$ to the second point $(2, 3)$ and $\mathbf{R} = x\mathbf{i} + y\mathbf{j}$ (20%)

4. A solid model was cut from a cylinder with radius
- r
- . Calculate the volume of the solid model. (20%)



front view, top view, side view

stereo view



5. A complex variable
- z
- is defined as
- $z = x + iy$
- . Hyperbolic sine and cosine are defined as
- $\sinh x = (e^x - e^{-x})/2$
- , and
- $\cosh x = (e^x + e^{-x})/2$
- , respectively.

(a) Show that the real part and the imaginary part of a complex function $(\sin z)$ are $(\sin x \cosh y)$ and $(\cos x \sinh y)$ respectively. (10%)(b) Determine the real part and imaginary part of a complex function $(\cos z)$. (10%)