國立臺灣科技大學
九十四學年度碩士班招生考試試題
系所組別：機械工程系碩士班丙組
科　　目：流體力學

◎ 總分100分

Note: A summary of required mathematical relations and governing equations is listed in page 3.

1. Answer the following questions: 每小題3分，共24分

   a) Define the streamline equation for Spherical coordinates.

      Note $x_1 = r, x_2 = \theta, x_3 = \phi; h_1 = 1, h_2 = r, h_3 = r\sin\theta$.

   b) What is the definition of bulk compressibility modulus (or modulus of elasticity)? Write down its expression for an idea gas undergoing an isothermal process.

   c) Describe the characteristics of *fluid statics*. What kinds of simplifications can be achieved on Newton 2^{nd} law for *fluid statics*?

   d) State the merits and disadvantages for integral and differential forms of governing equations,

   e) What is the continuum hypothesis? The major benefit for introducing the continuum assumption,

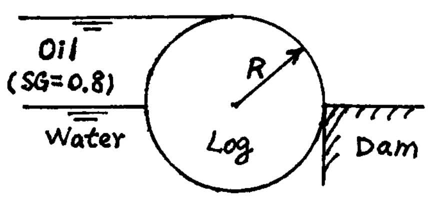
   f) Please write down the Reynolds Transport Theory and explain the physical meaning for each term. Also, please explain the reason why Reynolds Transport Theory is introduced.

   g) What are the "entrance length" and "fully developed flow"? For a fully developed flow in a pipe, please write down the thickness of its boundary layer.

   h) For flow past a plate, besides the Blausis solution, Von K'arm'an also developed the Momentum Integral Equation. What is the difference between them, and why he studied this problem again?

2. A cylindrical log of length = 6 m and R = 0.6 m, is in equilibrium, as shown. Calculate the force pushing it against the dam.

   ($\rho_{water} = 1000\ kg/m^3$, specific gravity of oil is 0.8) 本題 15 分



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3. The velocity field within a laminar boundary layer is given by

$$\vec{V} = \frac{AUy}{x^{1/2}}\hat{i} + \frac{AUy^2}{4x^{3/2}}\hat{j}$$

本題 16 分

In this expression, A=150 $m^{-1/2}$, and U=0.24 m/s is the freestream velocity.

(a) Is it an incompressible flow?

(b) Is it an irrotational flow?

(c) Determine the expression for stream function,

(d) Calculate the acceleration of a fluid particle at point (x, y) = (2m, 3.5m).

4. Air flows over a flat plate. The plate is of 2 m wide. Using the control volume shown, calculate the drag force acting on the flat plate. Outside the viscous region the velocity is uniform. ($\rho_{air} = 1.23\ kg/m^3$)

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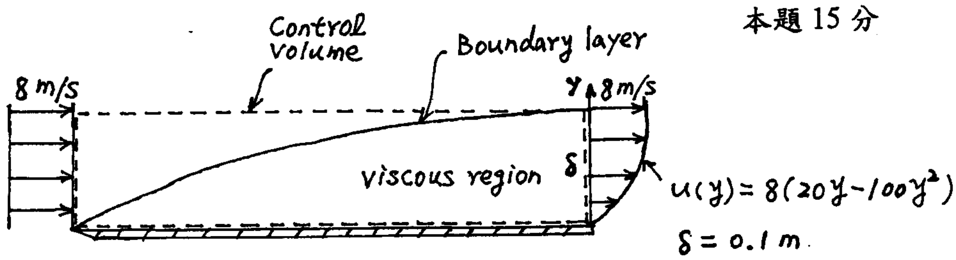

5. Two immiscible fluids are contained between infinite parallel plates. The plates are separated by distance 2h, and the two fluid layers are of equal thickness h; the dynamic viscosity of the upper fluid is three times that of the lower fluid. If the lower plate is stationary and the upper plate moves at constant speed U = 5 m/s, what is the velocity at the interface? Assume laminar flows, and that the pressure gradient in the direction of flow is zero.

本題 15 分

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6. For turbomachines, we are interested in the relation among the following six key parameters: 本題 15 分

(1) Volume flow rate, Q　　(2) Head (energy/mass), H
(3) Angular speed, N (rpm)　　(4) Characteristic diameter, D
(5) Fluid density, ρ　　(6) Fluid viscosity, μ

Please use MLTt system and select N, ρ, and D as repeating parameters to perform the nondimensional analysis. There will be 3 dimensionless groups generated as a result of this procedure. Please write them down and explain the corresponding physical meanings.

Summary of Equations

(1) Continuity Equation:

$$0=\frac{\partial}{\partial t}\int_{CV}\rho d\forall+\int_{CS}\rho\vec{V}\cdot d\vec{A}\,;\quad \nabla\cdot\rho\vec{V}+\frac{\partial\rho}{\partial t}=0\quad\text{or}\quad\frac{D\rho}{Dt}+\rho div\vec{V}=0$$

(2) Momentum Equation:

$$\vec{F}_S+\vec{F}_B-\int_{CV}\left[\vec{a}_{rf}+2\vec{\omega}\times\vec{V}_{xyz}+\vec{\omega}\times(\vec{\omega}\times\vec{r})+\dot{\vec{\omega}}\times\vec{r}\right]\rho d\forall$$
$$=\frac{\partial}{\partial t}\int_{CV}\vec{V}_{xyz}\rho d\forall+\int_{CS}\vec{V}_{xyz}\rho\vec{V}_{xyz}\cdot d\vec{A}$$

$$\rho\frac{D\vec{V}}{Dt}=-\nabla p+div\,\vec{\tau}+\rho\vec{g}$$

(3) The First Law of Thermodynamics:

$$\dot{Q}-\dot{W}_s-\dot{W}_{\text{shear}}-\dot{W}_{\text{other}}=\frac{\partial}{\partial t}\int_{CV}e\rho d\forall+\int_{CS}\left(u+pv+\frac{V^2}{2}+gz\right)\rho\vec{V}\cdot d\vec{A}$$

(4) Mathematics relations:

(a) Gradient Theorem

$$\iiint_V\nabla\phi d\tau=\oint\!\!\oint_S\phi\hat{n}\,dS$$

(b) Divergence Theorem

$$\iiint_V \text{div}\vec{A}d\tau=\oint\!\!\oint_S\vec{A}\cdot\hat{n}dS$$

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(c) Curl Theorem

$$\iiint_V \text{curl}\vec{A}\,d\tau = \oiint_S \hat{n} \times \vec{A}\,dS$$

(d) Gradient

$$\nabla\phi = \frac{\hat{e}_1}{h_1}\frac{\partial\phi}{\partial q^1} + \frac{\hat{e}_2}{h_2}\frac{\partial\phi}{\partial q^2} + \frac{\hat{e}_3}{h_3}\frac{\partial\phi}{\partial q^3}$$

(e) $$\text{div}\vec{A} = \frac{1}{h_1h_2h_3}\left[\frac{\partial}{\partial q^1}\left(h_2h_3\hat{A}_1\right) + \frac{\partial}{\partial q^2}\left(h_1h_3\hat{A}_2\right) + \frac{\partial}{\partial q^3}\left(h_1h_2\hat{A}_3\right)\right]$$

(f) $$\text{curl}\vec{A} = \frac{1}{h_1h_2h_3}\begin{vmatrix} h_1\hat{e}_1 & h_2\hat{e}_2 & h_3\hat{e}_3 \\ \dfrac{\partial}{\partial q^1} & \dfrac{\partial}{\partial q^2} & \dfrac{\partial}{\partial q^3} \\ h_1\hat{A}_1 & h_2\hat{A}_2 & h_3\hat{A}_3 \end{vmatrix}$$

(g) $$\vec{V}\cdot\nabla\vec{V} = \nabla\left(\frac{\vec{V}^2}{2}\right) - \vec{V}\times\text{curl}\vec{V}$$

(5) Scalar factors for orthogonal coordinates

Coordinates	Cartesians	Cylindrical	Spherical
h_1	$h_x = 1$	$h_r = 1$	$h_r = 1$
h_2	$h_y = 1$	$h_\theta = r$	$h_\theta = r$
h_3	$h_z = 1$	$h_z = 1$	$h_\phi = r\sin\theta$