國立臺灣科技大學
九十四學年度碩士班招生考試試題

系所組別：電子工程系碩士班乙一組
科　　目：工程數學

※總分為100分

1. (6%) Find the inverse of the block matrix given by

$$\begin{bmatrix} \mathbf{0} & \mathbf{I} \\ -\mathbf{I} & \mathbf{G} \end{bmatrix}$$

where $\mathbf{0}$ is an $n \times n$ zero matrix, $\mathbf{I}$ is an $n \times n$ identity matrix, and $\mathbf{G}$ is an $n \times n$ invertible matrix.

2. (16%) Let a 6×6 matrix $\mathbf{C}$ be defined as

$$\mathbf{C} = \mathbf{I} + \mathbf{J}$$

where $\mathbf{I}$ is a 6×6 identity matrix and

$$\mathbf{J} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

(a) (10%) Determine the nullspace of $\mathbf{C}$ and find its dimension, where the nullspace of $\mathbf{C}$ is define as $\{\mathbf{x}|\mathbf{C}\mathbf{x} = \mathbf{0}, \mathbf{x} \in \mathcal{R}^6\}$.

(b) (6%) Is it true that $\mathbf{C}\mathbf{x} = \mathbf{b}$ has a solution for *all* $\mathbf{b} \in \mathcal{R}^6$? Briefly explain your answer.

3. (16%) Suppose $\mathbf{A}$ is a 3×3 matrix with eigenvalues 1, 2, 3, then

(a) (3%) Is $\mathbf{A}$ diagonalizable? Briefly explain your answer.

(b) (3%) Determine the eigenvalues of $2\mathbf{A}^{-1} + \mathbf{I}$.

(c) (3%) Determine the determinant of $\mathbf{A} + \mathbf{I}$.

(d) (3%) Determine the determinant of $2(\mathbf{A}^T\mathbf{A})$.

(e) (4%) Determine $rank(\mathbf{A}^3)$.

4. (6%) Let T be a linear transformation which rotates every vector in $\mathcal{R}^2$ by 30^0 in the *counterclockwise* direction, then projects it on the x-axis. Determine the matrix representation of this linear transformation, i.e. if $T(\begin{bmatrix} x \\ y \end{bmatrix}) = \mathbf{B}\begin{bmatrix} x \\ y \end{bmatrix}$ for any vector $\begin{bmatrix} x \\ y \end{bmatrix} \in \mathcal{R}^2$, then $\mathbf{B}$=?

5. (6%) Find an orthonormal basis for the column space of

$$\mathbf{D} = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 4 & 0 \\ 1 & 4 & 6 \\ 1 & 4 & 6 \end{bmatrix}$$

90

國立臺灣科技大學
九十四學年度碩士班招生考試試題
系所組別：電子工程系碩士班乙一組
科　　目：工程數學

(6) Let $p(x)=\frac{1}{2}e^{-|x|}$, $-\infty<x<\infty$

Find E[min(|x|,1)] . (11%)

(7) Let $f_{XY}(x,y)=\begin{cases}A, 0<x<1, 0<x<y\\ 0, otherwise\end{cases}$, where A is a constant .

Find the correlation ρ_{XY} for X and Y (13%)

(8) X , Y are independent random variables with Binomial distribution .
where X~B(n_1, p) , Y~B(n_2, p) . Express P{ X=k | X+Y=n } in terms of

$C^{n}_{n_1+n_2}, C^{k}_{n_1}, C^{n-k}_{n_2}$ (13%)

(9) X , Y are independent random variables and obey the same distribution

$p(x)=\begin{cases}e^{-x}, x>0\\ 0, x\le 0\end{cases}$ Let $U=X+Y, V=\frac{X}{Y}$

prove that U , V are independent (13%)

91