國立臺灣科技大學

九十四學年度碩士班招生考試試題

系所組別：電子工程系碩士班乙二組

科　　目：工程數學

注意: 答案卷內,請依題號順序撰寫答案 (例如:第 1 頁寫第 1 題,第 2 頁寫第 2 題,...) 必須分為100分

1. Find the general solution of $x^2y+y^3+xy^2y'=0$; y(1)=1 (10%)

2. Find the general solution of y''+y'-2y=f(t); y(0)=0,y'(0)=0 with

$$f(t)=\begin{cases}0 & for \quad 0\le t<6\\ 2 & for \quad t\ge 6\end{cases}$$

(15%)

3. Find the general solution of $x^2y''+3xy'+y=x^2+2x+3$ (10%)

4. Let u(x,t)=X(x)T(t). Write a solution of the following boundary problem.

$\frac{\partial u}{\partial t}=k\frac{\partial^2 u}{\partial x^2}$ for 0<x<L, t>0

u(0,t)=u(L,t)=0 for t≧0

u(x,0)=1 for 0≦x≦L

(15%)

5.(a)Let A be an $n\times n$ matrix with detA≠0, and adjA denotes the adjoint matrix of A. λ is one eigenvalue of A. Find the eigenvalue of $(adjA)^2+I_{n\times n}$. **(5%)** (b) Let A be an $n\times n$ matrix and $A^2+2A+2I=0$. For any real k, is $A+kI$ invertible? If yes, find the inverse matrix in terms of A and k. **(5%)** (c) A is an $n\times n$ real and symmetric matrix. If k is an integer and k≥2. If $A^k=0$, find A. **(5%)**

6. (a) If vectors $X_1,X_2,\cdots,X_n$ are linearly independent for n > 2 and n can be an odd or even number. Determine $(X_1+X_2),(X_2+X_3),(X_3+X_4),\ldots(X_n+X_1)$ to be linearly dependent or independent. **(10%)** (b) If three column vectors of size 3 v_1,v_2,v_3 are linearly independent. Find rref$[v_1:v_2:v_3]$, where rref denotes the reduced row-echelon form.**(5%)**

7. (a)In order to make the geometric matrix series $I+A+A^2+\cdots$ converge, what kind of the eigenvalue of A is? **(5%)** (b)Find the sum of the matrix series in part (a). **(5%)**

8. Decompose $\begin{bmatrix}5 & -5 & 10 & 0 & 5\\ -3 & 3 & 2 & 2 & 1\\ -2 & 2 & 0 & -1 & 0\\ 1 & -1 & 10 & 2 & 5\end{bmatrix}$ into LU,where L and U denote lower and upper triangular matrices, respectively, and U should have leading 1 in each row. **(10%)**

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