

國立臺灣科技大學
九十四學年度碩士班招生考試試題

系所組別：電子工程系碩士班丙組
科 目：工程數學

注意：答案卷內，請依題號順序撰寫答案（例如：第 1 頁寫第 1 題，第 2 頁寫第 2 題，...） * 總分為 100 分

1. Find the general solution of $x^2y+y^3+xy^2y'=0$; $y(1)=1$ (10%)

2. Find the general solution of $y''+y'-2y=f(t)$; $y(0)=0, y'(0)=0$ with

$$f(t)=\begin{cases} 0 & \text{for } 0 \leq t < 6 \\ 2 & \text{for } t \geq 6 \end{cases}$$

(15%)

3. Find the general solution of $x^2y''+3xy'+y=x^2+2x+3$ (10%)

4. Let $u(x,t)=X(x)T(t)$. Write a solution of the following boundary problem.

$$\frac{\partial u}{\partial t} = k \frac{\partial^2 u}{\partial x^2} \quad \text{for } 0 < x < L, t > 0$$

$$u(0,t)=u(L,t)=0 \quad \text{for } t \geq 0$$

$$u(x,0)=1 \quad \text{for } 0 \leq x \leq L$$

(15%)

5.(a) Let A be an $n \times n$ matrix with $\det A \neq 0$, and $\text{adj}A$ denotes the adjoint matrix of A . λ is one

eigenvalue of A . Find the eigenvalue of $(\text{adj}A)^2 + I_{n \times n}$. (5%) (b) Let A be an $n \times n$ matrix

and $A^2 + 2A + 2I = 0$. For any real k , is $A + kI$ invertible? If yes, find the inverse matrix in terms of A and k . (5%) (c) A is an $n \times n$ real and symmetric matrix. If k is an integer and

$k \geq 2$. If $A^k = 0$, find A . (5%)

6. (a) If vectors X_1, X_2, \dots, X_n are linearly independent for $n > 2$ and n can be an odd or even

number. Determine $(X_1 + X_2), (X_2 + X_3), (X_3 + X_4), \dots, (X_n + X_1)$ to be linearly dependent

or independent. (10%) (b) If three column vectors of size 3 v_1, v_2, v_3 are linearly

independent. Find $\text{rref}[v_1 : v_2 : v_3]$, where rref denotes the reduced row-echelon form. (5%)

7. (a) In order to make the geometric matrix series $I + A + A^2 + \dots$ converge, what kind of the

eigenvalue of A is? (5%) (b) Find the sum of the matrix series in part (a). (5%)

8. Decompose $\begin{bmatrix} 5 & -5 & 10 & 0 & 5 \\ -3 & 3 & 2 & 2 & 1 \\ -2 & 2 & 0 & -1 & 0 \\ 1 & -1 & 10 & 2 & 5 \end{bmatrix}$ into LU, where L and U denote lower and upper triangular

matrices, respectively, and U should have leading 1 in each row. (10%)

