

國立臺灣科技大學
九十四學年度碩士班招生考試試題

系所組別：電機工程系碩士班乙一組
科 目：微分方程及線性代數

總分 100 分

1. (15%) Find the general solution of the following equation

$$y''(t) + 16y(t) = f(t)$$

$$\text{where } f(t) = \begin{cases} \cos 4t, & 0 \leq t < \pi \\ 0, & t \geq \pi \end{cases} \text{ and } y(0) = 0, \quad y'(0) = 1$$

2. (15%) Find the general solution of

$$y''' + 2y'' + 9y' + 18y = -7 \cos(3x)$$

3. (10%) Find the general solution of the differential equation

$$(2y^2 - 9xy)dx + (3xy - 6x^2)dy = 0$$

4. (10%) Solve the following equation

$$x^2 y'' - xy' + y = \ln x$$

5. (15%) Consider the matrix

$$A_t = \begin{pmatrix} 3 & t & -1 \\ t & 4 & 0 \\ -1 & 0 & 3 \end{pmatrix}$$

- a) Calculate $\det(A_t)$
b) Find all possible $t \in \mathfrak{R}$ for which A_t is invertible?
c) Find $t \in \mathfrak{R}$ such that $(1, 0, 1)^T$ is an eigenvector of A_t associated to the eigenvalue 2.

6. (15%) Diagonalize the matrix $A = \begin{pmatrix} -7 & -16 & 4 \\ 6 & 13 & -2 \\ 12 & 16 & 1 \end{pmatrix}$, using the information that

the eigenvalues of A are $\lambda = -3, 5$

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7. (10%) Determine that the transformation T defined by $T(X_1, X_2) = (2X_1 - 3X_2, X_1 + 4, 5X_2)$ is linear or not. Prove it.

8. (10%) Explain how you can tell that $P = \frac{1}{31} \begin{pmatrix} 2 & 5 & 2 & 2 & 5 \\ 5 & 28 & 5 & 5 & -3 \\ 2 & 5 & 2 & 2 & 5 \\ 2 & 5 & 2 & 2 & 5 \\ 5 & -3 & 5 & 5 & 28 \end{pmatrix}$ is a

projection matrix?

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