國立臺灣科技大學
九十四學年度碩士班招生考試試題
系所組別：電機工程系碩士班丙一組、電機工程系碩士班丙二組
科　　目：線性代數與機率

※總分100分

1. (15%) L: $P_3(R) \rightarrow P_3(R)$ is a linear transformation. Define $L(f(x)) = 2x\dfrac{df(x)}{dx} + \dfrac{d^3 f(x)}{dx^3}$ where $f(x) \in P_3(R)$. Let $\beta = \{1, x, x^2, x^3\}$ and $\beta_o = \{1-x, 1+x, x^2+x^3, x^3\}$ be an ordered basis of $P_3(R)$.

   (a) (5%) Find the matrix A representing L with respect to β (i.e. $A = [L]_\beta$).

   (b) (5%) Find the matrix B representing L with respect to β_o (i.e. $B = [L]_{\beta_o}$).

   (c) (5%) Find the matrix S such that $B=SAS^{-1}$ (i.e. $S = [I_v]_{\beta_o}^{\beta}$).

2. (10%) Let V be a real inner product space, and $\boldsymbol{u}$, $\boldsymbol{v}$ be two nonzero elements in V. Suppose that $\boldsymbol{u}$, $\boldsymbol{v}$ are orthogonal. Show that the set $\{\boldsymbol{u}, \boldsymbol{v}\}$ is linearly independent.

3. (15%) Let T : $R_3 \rightarrow R_3$ be a linear operator represented by the matrix

$$A = \begin{bmatrix} 0 & -3 & -3 \\ 1 & 4 & 3 \\ 0 & 0 & 1 \end{bmatrix}$$

   (a) (3%) Find the determinant of A and A^{-1}.

   (b) (4%) Find the eigenvalues and corresponding eigenvectors of A.

   (c) (4%) Find a matrix P such that $P^{-1}AP=D$, where D is a diagonal matrix.

   (d) (4%) Find the eigenvalues of matrix A^3+2A^2+5A.

4. (10%) Let $V = C^3$, and $S = \{(1, i, 0), (2, 1, -i)\}$, find $S^\perp$ such that $S^\perp = \{v \in V \mid \langle v, s \rangle = 0, \forall s \in S\}$.

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5. (10%)
(1) Find the probability that the sum of the outcomes of five tosses of a die is 9.
(2) If P(A)=0.2 and P(B)=0.3, find $P(A \cup B)$ if A and B are independent.
(3) Let $X = \cos\Theta$ and $Y = \sin\Theta$, where Θ is an angle that is uniformly distributed in the interval $(0, 2\pi)$. Find the conditional probability density function (pdf) $f(y \mid x)$.

6. (15%) A sequential experiment involves repeatedly drawing a ball from one of two urns, noting the number on the ball, and replacing the ball in its urn. Urn 0 contains a ball with the number 1 and two balls with the number 0, and urn 1 contains five balls with the number 1 and one ball with the number 0. The urn from which the first draw is made is selected at random by flipping a fair coin. Urn 0 is used if the outcome is heads and urn 1 if the outcome is tails. Thereafter the urn used in a subexperiment corresponds to the number on the ball selected in the previous subexperiment. Let $p_0(n)$ and $p_1(n)$ be the probabilities that urn 0 or urn 1 is used in the n th subexperiment.
(1) Find $p_0(n)$ and $p_1(n)$.
(2) What are the urn probabilities as n approaches infinity?

7. (15%) The random variables X and Y have the joint pdf

$$f_{X,Y}(x,y) = 2e^{-(x+y)}, \quad 0 \le y \le x < \infty.$$

(1) Find the marginal pdf's $f_X(x)$ and $f_Y(y)$.
(2) Find the pdf of $Z = X + Y$.

8. (10%) A limiter $Y = g(X)$ is shown in Fig. 1.

Find the pdf of Y if X has a Laplacian pdf, i.e., $f_X(x) = \frac{\beta}{2}e^{-\beta|x|}, \ -\infty < x < \infty$.

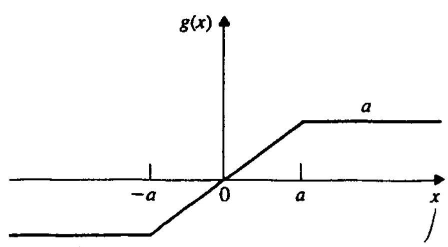

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Fig. 1