515001

國立臺灣科技大學
九十四學年度碩士班招生考試試題
系所組別：資訊工程系碩士班
科　　目：計算機數學

總分 100 分

1. (8%) There are several proof techniques for proving theorems. What are the methods of an "indirect proof" and a proof by contradiction? Describe and compare them.

2. (12%) Let n be a positive integer. Show that any $2^n \times 2^n$ chessboard with one square removed can be tiled using L-shaped pieces, where each piece covers three squares at a time, as shown in Figure 1.

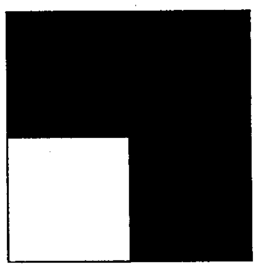

Figure 1.　An L-shaped piece.

3. (a) (3%) Find a recurrence relation for the number of ways to climb n stairs if the person climbing the stairs can take one, two, or three stairs at a time.
   (b) (3%) What are the initial conditions?
   (c) (4%) How many ways can this person climb a flight of eight stairs?

4. (10%) Show that an edge in a simple graph is a cut edge if and only if this edge is not part of any simple circuit in the graph.

5. Let $T: R^2 \rightarrow R^2$ be the linear transformation defined by $T(\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}) = \begin{bmatrix} x_1 + x_2 \\ -2x_1 + 4x_2 \end{bmatrix}$, and let the basis $B = \{\begin{bmatrix} 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \end{bmatrix}\}$.
   (a) (5%) Find the matrix for T with respect to the basis B.
   (b) (5%) Let $x = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$. Find $[x]_B$ and $[T(x)]_B$.

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6. (10%) Let $A = \begin{bmatrix} 2 & 0 \\ 0 & 2 \\ 1 & -1 \end{bmatrix}$ be a matrix in $R^{3\times 2}$ and $v = \begin{bmatrix} 1 \\ 4 \\ 0 \end{bmatrix}$ be a vector in R^3. Find the projection of v onto the column space of matrix A.

7. Answer the following "Yes" or "No" questions with your justification.
Warning: *You will not get any point without the justification.*
   (a) (3%) Let S and T be two subspaces of R^n. Then $W = S \cup T$ is a subspace.
   (b) (3%) Let A and B be two symmetric and invertible matrices in $R^{n\times n}$. Then $AB = BA$.
   (c) (4%) Let $A = \begin{bmatrix} 0.75 & -0.25 \\ -0.25 & 0.75 \end{bmatrix}$. Then $\lim_{n\to\infty} A^n = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$.

8. (10%) For a continuous non-negative random variable X with CDF $F(x)$, prove that $E[X] = \int_0^\infty (1 - F(x))dx$.

9. (10%) Assume that medical science has a test for cancer which is 90% accurate for both those who do and those who do not have cancer. Also assume that 5% of the population tested actually has cancer. If an individual takes this cancer test and the result is positive, what is the probability the person really does have cancer?

10.(10%) If Z is a random variable with the standard normal distribution, find the probability density function of Z^2.

148