國立臺灣科技大學

九十四學年度碩士班招生考試試題

系所組別：工業管理系碩士班甲組、乙組、丙組、丁組

科　　目：統計學

(Total 100 points.) *There are 4 problems in this exam. Show intermediate steps and formulas for partial credit. You must explain how you compute your results or answers for full credit.*

1. (25 points)

Let X have the probability density function (p.d.f.), $f(x) = k \cdot |\cos(x)|$ for $-\pi \le x \le \pi$, where $|\cdot|$ means the absolute value.

(a) (5 points) Determine the appropriate value of k.

(b) (5 points) Compute the probability if $X \ge -\frac{\pi}{2}$.

(c) (7 points) Compute the value of $E(X \mid X \ge -\frac{\pi}{2})$.

(d) (8 points) Determine the value of c if X has the p.d.f., $f(x) = c \cdot |\cos(x)|$ for $d_1\pi \le x \le d_2\pi$, where d_1 and d_2 are any integers with $d_2 > d_1$.

2. (25 points)

Let $X_1, X_2, ..., X_n$ be a random sample from the following uniformly distribution.

$$f(x) = \frac{1}{\theta}, \quad \theta \le x \le 2\theta, \quad \theta > 0.$$

(a) (5 points) Write down the likelihood function clearly.

(b) (5 points) Show that $\frac{1}{2}X_{(1)} + \frac{1}{4}X_{(n)}$ is one maximum likelihood estimator of θ if $X_{(1)} \le X_{(2)} \le ... \le X_{(n)}$ represent the order statistics of the random sample.

(c) (7 points) Find the density function of $X_{(1)}$.

(d) (8 points) Utilize the variable $X_{(1)}$ to construct the 95% equal-tail confidence interval for the parameter θ.

3. (25 points)

A company would like to determine whether it would be profitable to establish a new service for their customers. The company believes that there are four possible levels of demand for this service:

D_1: very low demand; 1% of the customers would use the service.

D_2: low demand; 5% of the customers would use the service.

D_3: moderate demand; 10% of the customers would use the service.

D_4: high demand; 25% of the customers would use the service.

On the basis of past experience, the company assigns the following prior probabilities to these demand levels: $P(D_1) = 0.2, P(D_2) = 0.5, P(D_3) = 0.2$, and $P(D_4) = 0.1$. Before the decision is made, a random sample of 5 customers is selected and only 1 would use the service.

(a) (5 points) Given that the true demand is moderate, what is the probability that the above random sample would occur?

(b) (10 points) Under the prior probabilities, what is the probability that the above random sample would occur?

(c) (10 points) Revise the prior probabilities using this sample information.

4. (25 points)

The NTUST Company is interested in computing a pooled estimate of the variance of two populations. For the first population, the following data are collected; $n_1 = 8, \bar{X}_1 = 10, S_1^2 = 40.84$. Similarly, for the second population, we have $n_2 = 10, \bar{X}_2 = 14, S_2^2 = 86.50$.

(a) (5 points) What is the 95% confidence interval for the mean of the second population, μ_2? (two-sided)

(b) (10 points) Pooling is justified if it appears reasonable to assume that the two populations have equal variances. Is pooling appropriate in this situation (test with $\alpha = 0.1$)?

(c) (10 points) Test if μ_1 equals to μ_2 (test with $\alpha = 0.05$)?

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t DISTRIBUTION

Entries in the table give *t* values for an area or probability in the upper tail of the *t* distribution. For example, with 10 degrees of freedom and a .05 area in the upper tail, $t_{.05} = 1.812$.

Degrees of Freedom	Area in Upper Tail				
	.10	.05	.025	.01	.005
1	3.078	6.314	12.706	31.821	63.657
2	1.886	2.920	4.303	6.965	9.925
3	1.638	2.353	3.182	4.541	5.841
4	1.533	2.132	2.776	3.747	4.604
5	1.476	2.015	2.571	3.365	4.032
6	1.440	1.943	2.447	3.143	3.707
7	1.415	1.895	2.365	2.998	3.499
8	1.397	1.860	2.306	2.896	3.355
9	1.383	1.833	2.262	2.821	3.250
10	1.372	1.812	2.228	2.764	3.169
11	1.363	1.796	2.201	2.718	3.106
12	1.356	1.782	2.179	2.681	3.055
13	1.350	1.771	2.160	2.650	3.012
14	1.345	1.761	2.145	2.624	2.977
15	1.341	1.753	2.131	2.602	2.947
16	1.337	1.746	2.120	2.583	2.921
17	1.333	1.740	2.110	2.567	2.898
18	1.330	1.734	2.101	2.552	2.878
19	1.328	1.729	2.093	2.539	2.861
20	1.325	1.725	2.086	2.528	2.845
21	1.323	1.721	2.080	2.518	2.831
22	1.321	1.717	2.074	2.508	2.819
23	1.319	1.714	2.069	2.500	2.807
24	1.318	1.711	2.064	2.492	2.797
25	1.316	1.708	2.060	2.485	2.787
26	1.315	1.706	2.056	2.479	2.779
27	1.314	1.703	2.052	2.473	2.771
28	1.313	1.701	2.048	2.467	2.763
29	1.311	1.699	2.045	2.462	2.756
30	1.310	1.697	2.042	2.457	2.750
40	1.303	1.684	2.021	2.423	2.704
60	1.296	1.671	2.000	2.390	2.660
120	1.289	1.658	1.980	2.358	2.617
∞	1.282	1.645	1.960	2.326	2.576

F DISTRIBUTION

Entries in the table give F_α values, where α is the area or probability in the upper tail of the *F* distribution. For example, with 12 numerator degress of freedom, 15 denominator degrees of freedom, and a .05 area in the upper tail, $F_{.05} = 2.48$.

Table of $F_{.05}$ Values

Denominator Degrees of Freedom	Numerator Degrees of Freedom																		
	1	2	3	4	5	6	7	8	9	10	12	15	20	24	30	40	60	120	∞
1	161.4	199.5	215.7	224.6	230.2	234.0	236.8	238.9	240.5	241.9	243.9	245.9	248.0	249.1	250.1	251.1	252.2	253.3	254.3
2	18.51	19.00	19.16	19.25	19.30	19.33	19.35	19.37	19.38	19.40	19.41	19.43	19.45	19.45	19.46	19.47	19.48	19.49	19.50
3	10.13	9.55	9.28	9.12	9.01	8.94	8.89	8.85	8.81	8.79	8.74	8.70	8.66	8.64	8.62	8.59	8.57	8.55	8.53
4	7.71	6.94	6.59	6.39	6.26	6.16	6.09	6.04	6.00	5.96	5.91	5.86	5.80	5.77	5.75	5.72	5.69	5.66	5.63
5	6.61	5.79	5.41	5.19	5.05	4.95	4.88	4.82	4.77	4.74	4.68	4.62	4.56	4.53	4.50	4.46	4.43	4.40	4.36
6	5.99	5.14	4.76	4.53	4.39	4.28	4.21	4.15	4.10	4.06	4.00	3.94	3.87	3.84	3.81	3.77	3.74	3.70	3.67
7	5.59	4.74	4.35	4.12	3.97	3.87	3.79	3.73	3.68	3.64	3.57	3.51	3.44	3.41	3.38	3.34	3.30	3.27	3.23
8	5.32	4.46	4.07	3.84	3.69	3.58	3.50	3.44	3.39	3.35	3.28	3.22	3.15	3.12	3.08	3.04	3.01	2.97	2.93
9	5.12	4.26	3.86	3.63	3.48	3.37	3.29	3.23	3.18	3.14	3.07	3.01	2.94	2.90	2.86	2.83	2.79	2.75	2.71
10	4.96	4.10	3.71	3.48	3.33	3.22	3.14	3.07	3.02	2.98	2.91	2.85	2.77	2.74	2.70	2.66	2.62	2.58	2.54
11	4.84	3.98	3.59	3.36	3.20	3.09	3.01	2.95	2.90	2.85	2.79	2.72	2.65	2.61	2.57	2.53	2.49	2.45	2.40
12	4.75	3.89	3.49	3.26	3.11	3.00	2.91	2.85	2.80	2.75	2.69	2.62	2.54	2.51	2.47	2.43	2.38	2.34	2.30
13	4.67	3.81	3.41	3.18	3.03	2.92	2.83	2.77	2.71	2.67	2.60	2.53	2.46	2.42	2.38	2.34	2.30	2.25	2.21
14	4.60	3.74	3.34	3.11	2.96	2.85	2.76	2.70	2.65	2.60	2.53	2.46	2.39	2.35	2.31	2.27	2.22	2.18	2.13