

國立臺灣科技大學
九十四學年度碩士班招生考試試題

系所組別：資訊管理系碩士班甲組
科 目：離散數學

本試卷共七大題，滿分共計 100 分

一、(10 points) Find all solutions to the system of congruence.

$$x \equiv 2 \pmod{3}$$

$$x \equiv 3 \pmod{4}$$

$$x \equiv 5 \pmod{7}$$

二、(a) (7 points) Find the solution to the recurrence relation

$$a_n = 8a_{n-1} + 9a_{n-2} \text{ if } a_0 = 3 \text{ and } a_1 = 7.$$

(b) (8 points) Find the solution to the recurrence relation

$$a_n = a_{n-1} + n \text{ if } a_1 = 1$$

三、Find a closed form for the generating function for the sequence $\{a_n\}$, where

(a) (7 points) $a_n = 4n + 2$ for $n = 0, 1, 2, \dots$

(b) (8 points) $a_n = \frac{2}{(n-1)!}$ for $n = 1, 2, 3, \dots$

四、(10 points) Let f and g be functions. If f and $f \circ g$ are one-to-one, does it follow that g is one-to-one? If it is, prove your answer; otherwise give a counterexample.

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五、Consider the 8×6 grid shown in Fig. 1.

- (a) (10 points) How many different rectangles, which area is equal to or larger than 4, does this grid contain? [For example, there is a rectangle with corners $(5,0)$, $(8,0)$, $(8,2)$, $(5,2)$, and its area is 6].
- (b) (10 points) How many ways can a path from the origin to the point $(8,6)$ if the moves that are allowed are of the forms, $R: (x, y) \rightarrow (x+1, y)$ and $U: (x, y) \rightarrow (x, y+1)$, and under the below constraints :
- does not use the path from $(4,4)$ to $(5,4)$ to $(6,4)$ to $(6,5)$, and
 - must use the path from $(2,2)$ to $(3,2)$

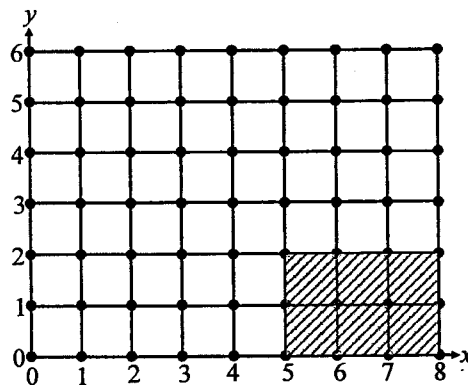


Fig. 1

六、For graph $K_{4,4}$ in Fig. 2,

- (a) (10 points) How many different one-to-one mapping it has?
- (b) (10 points) How many spanning trees it has?

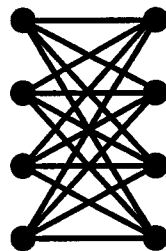


Fig. 2

七、(10 points) For all $n \in \mathbb{Z}^+$ (正整數), show that if $n \geq 24$, then n can be written as a sum of 5's and/or 7's.

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