國立台灣科技大學九十五學年度碩士班招生試題

系所組別： 自動化及控制研究所碩士班甲組、丙組

科　　目： 工程數學

總分為100分，題號請標示清楚。

1. Solve the following differential equations:

(1) $x^2y'+xy-y^2=x^2$ **(10%)**

(2) $x^2y''-4xy'+6y=x^2+2x$ **(10%)**

2. Use the Laplace Transform method to solve the following system with initial conditions. (15%)

$$x'+2x-y'=0$$

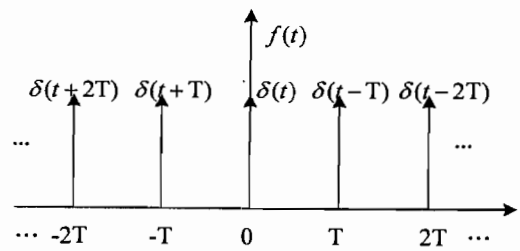
$$x'+x+y=t^2$$

where x(0) =0 , y(0)=0 .

3. Write the Taylor series for 1/(1+x) about 1 and find the first **five** nonzero parameters of the power series solution. For what values of x will this Taylor series converge.(15%)

4. The function f(t) is

$$f(t)=\sum_{k=-\infty}^{\infty}\delta(t-kT),\quad k=0,\pm1,\pm2...$$



where $\delta(t)$ is the Dirac delta function and

T indicates the period of f(t)

(a) Please solve the complex Fourier series of f(t). **(8%)**

(b) Please solve the Fourier transform of f(t). **(7%)**

5. Evaluate the function I using the residue theorem **(15%)**

$$I=\int_0^{2\pi}\frac{\cos 2\theta}{1-2p\cos\theta+p^2}\,d\theta,\ \text{where } p>1$$

6. (a) Please use the Caley-Hamilton theorem to prove

$$f(\mathbf{A})=r_1\mathbf{A}_{n-1}+r_2\mathbf{A}_{n-2}+\cdots+r_{n-1}\mathbf{A}+r_n\mathbf{I}$$

where f(x) is an analytic scalar function of a scalar x; **A** is an $n\times n$ matrix; **I** is the $n\times n$ identity matrix. **(10%)**

(b) Let $\mathbf{A}=\begin{bmatrix}0 & i\\ -i & 0\end{bmatrix}$, please find e^{At}. **(10%)**