

## 國立台灣科技大學九十五學年度碩士班招生試題

系所組別：機械工程系碩士班甲組、乙組、丙組、丁組、戊組

科目：工程數學

總分 100 分

1. Solve the ordinary differential equation  $(1+x^2)y'' - 2xy' + 2y = 0$ . (15%)

2. Solve the simultaneous equations 
$$\begin{cases} y' + y + 3 \int_0^t z dt = \cos t + 3 \sin t \\ 2y' + 3z' + 6z = 0 \end{cases}$$
 with the initial conditions  $y(0) = -3$  and  $z(0) = 2$ . (15%)

3. Use an orthogonal transformation to transform the real quadratic form

$$Q = x_1^2 + x_2^2 + x_3^2 + 2x_1x_2 + 2x_2x_3 + 2x_1x_3$$

into its canonical form. Write down the canonical form and the transformation matrix. (20%)

4. Find the value of the integral  $\iint_S (\nabla \times V) \cdot n dA$  over the part of the unit sphere  $x^2 + y^2 + z^2 = 1$  above the  $xy$  plane, where  $V = y i$ . (15%)

5. Find the steady-state temperature distribution of the boundary value problem

$$\frac{\partial T}{\partial t} = \frac{\partial^2 T}{\partial x^2} \quad 0 \leq x \leq L, \quad T(0, t) = T_1, \quad T(L, t) = T_2. \quad (15\%)$$

6. (a) Find the region of convergence for the Laurent series of the function

$$f(z) = \frac{1}{z^4 + 4} \text{ in powers of } z-1 \text{ which converges when } z=i. \quad (10\%)$$

(b) Find the first three nonzero terms in the Laurent series of  $f(z) = \csc z$

$$\text{which is valid when } 0 < |z| < \pi. \quad (10\%)$$

