

## 國立台灣科技大學九十五學年度碩士班招生試題

系所組別：化學工程系碩士班

科目：工程數學

總分 100 分，請詳列計算過程並依序作答，否則酌予扣分。

1. Solve the differential equation:  $\frac{dy}{dx} = \frac{3x+y-1}{6x+2y-3}$  (10%)

2. Solve the differential equation:  $x \frac{d^2y}{dx^2} + x^2 \frac{dy}{dx} + 2xy = 0$  for  $x > 0$  (15%)

3. Compute the inverse Laplace transform:  $L^{-1} \left[ \frac{3s+5}{s^2+4s+8} \right]$  (10%)

4. With a surface  $\varphi(x, y, z) = x^2 - y^2 - z^2 = 0$  at the point  $(1, 1, 0)$ , find:

- (i) Direction of its maximum rate of change
- (ii) Equation of the tangent plane
- (iii) Equation of the normal line (15%)

5. Let  $F = f_1 \mathbf{i} + f_2 \mathbf{j} + f_3 \mathbf{k}$  and  $G = g_1 \mathbf{i} + g_2 \mathbf{j} + g_3 \mathbf{k}$  are vector fields. Show that

$$\nabla(F \cdot G) = (F \cdot \nabla)G + (G \cdot \nabla)F + F \times (\nabla \times G) + G \times (\nabla \times F)$$

Where  $\nabla = \frac{\partial}{\partial x} \mathbf{i} + \frac{\partial}{\partial y} \mathbf{j} + \frac{\partial}{\partial z} \mathbf{k}$  (15%)

6. Solve the system

$$\begin{pmatrix} y_1 \\ y_2 \end{pmatrix}' = \begin{pmatrix} 1 & -10 \\ -1 & 4 \end{pmatrix} \begin{pmatrix} y_1 \\ y_2 \end{pmatrix} + \begin{pmatrix} e^t \\ \sin(t) \end{pmatrix} \quad (15\%)$$

7. Solve the PDE

$$\frac{\partial T}{\partial t} = k \left( \frac{\partial^2 T}{\partial r^2} + \frac{1}{r} \frac{\partial T}{\partial r} \right) \text{ for } 0 \leq r < R, t > 0$$

$$T(R, t) = 0 \text{ and } T(0, t) = \text{finite for } t > 0$$

$$T(r, 0) = f(r) \text{ for } 0 \leq r < R \quad (20\%)$$

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