

## 國立台灣科技大學九十五學年度碩士班招生試題

系所組別：電子工程系碩士班乙一組

科目：工程數學

總分 100 分

1. Briefly answer the following questions. You will not get any credit if only the answer is given.

(a) (5%) Consider a  $3 \times 3$  system of linear equations  $Ax = b$ , where

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 5 & 8 \\ 3 & 5 & 7 \end{bmatrix} \quad \text{and} \quad b = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$$

Determine the condition on  $b_1$ ,  $b_2$ , and  $b_3$  such that  $Ax = b$  does not have a solution.(b) (5%) Let  $A$  be an  $n \times n$  matrix with rank  $r$ , then which of the following matrices also has(have) rank  $r$ ?

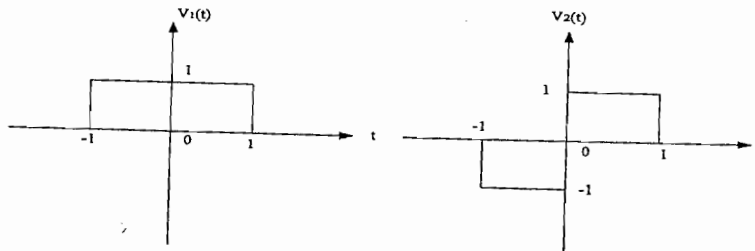
$$3A^T, [2A \quad 3A], \begin{bmatrix} A \\ A \end{bmatrix}, \begin{bmatrix} A & A \\ A & A \end{bmatrix}$$

(c) (5%) PLOT  $\text{span}\left(\begin{bmatrix} 1 \\ 1 \end{bmatrix}\right) \cup \text{span}\left(\begin{bmatrix} 0 \\ 1 \end{bmatrix}\right)$ .2. Let  $P_n$  denote the set of all polynomials of degree less than  $n$ . Now, consider two subspaces  $V$  and  $W$  of  $P_{10}$  which are given by

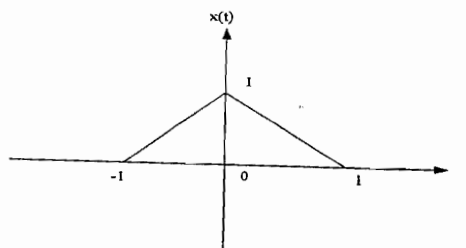
$$V = \{p(x) : p(x) = x^9 p(x^{-1})\} \quad \text{and} \quad W = \{q(x) : q(x) = q(-x)\}$$

(a) (5%) Determine  $\dim(V)$ .(b) (5%) Determine  $\dim(V \cap W)$ .3. (a) (5%) Suppose that  $A = \begin{bmatrix} 6 & -4 \\ \alpha & \beta \end{bmatrix}$ , then determine  $\alpha$  and  $\beta$  such that  $A$  has eigenvectors

$$x_1 = \begin{bmatrix} 4 \\ 3 \end{bmatrix} \quad \text{and} \quad x_2 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}.$$

(b) (5%) (a) continued. Determine  $\text{Trace}(A^3)$ , where  $\text{Trace}(C)$  denotes the summation of all of the diagonal elements of  $C$ , i.e.  $\text{Trace}(C) = \sum_{i=1}^n |C|_{i,i}$ .(c) (5%) Consider another  $2 \times 2$  matrix  $B$  with the same eigenvectors  $x_1$  and  $x_2$  as (a) and with respective eigenvalues  $\lambda_1 = 1$  and  $\lambda_2 = 0$ . Determine  $B^{10}$ .4. (10%) Consider a communication system which transmits the message  $\gamma$  and  $\eta$  through a linear combination with two known waveforms  $v_1(t)$  and  $v_2(t)$  by  $\gamma v_1(t) + \eta v_2(t)$ , where  $\gamma$  and  $\eta$  are real numbers, and  $v_1(t)$  and  $v_2(t)$  are given by

The receiver receives  $x(t)$  and determines the transmitted  $\gamma$  and  $\eta$  by choosing  $\gamma$  and  $\eta$  which minimize  $\|x(t) - (\gamma v_1(t) + \eta v_2(t))\|$ , where  $\|y(t)\| = \sqrt{\langle y(t), y(t) \rangle}$  with  $\langle y(t), z(t) \rangle = \int_{-1}^1 y(t)z(t) dt$ . Now suppose that the received signal  $x(t)$  is as given below. Determine the transmitted  $\gamma$  and  $\eta$ .



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5.  $X, Y$  are continuous random variables, prove  $E\{E\{Y|X\}\} = E\{Y\}$ .

(10分)

6.  $X$  is a random variable,  $a, n$  are two arbitrary numbers prove that

$$P\{|X-a| \geq \varepsilon\} \leq \frac{E\{|X-a|^n\}}{\varepsilon^n} \quad (10分)$$

7. In the circuits of Fig.1,  $R = \frac{1}{4}\Omega$  is a constant resistance and voltage source  $E$  is a random variable of Gaussian with zero mean and variance  $\sigma^2$ . Find the cumulative distribution function for the power loaded on the resistor. (15分)

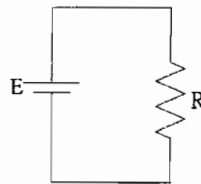


Fig.1

8.  $Z = \sqrt{X^2 + Y^2}$ ,  $W = \frac{X}{Y}$ ,  $X, Y$  are random variables with

$$f_{XY}(x, y) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x^2+y^2)}{2\sigma^2}} \quad \text{find } f_Z(z) \text{ and } f_W(w). \quad (15分)$$

