國立台灣科技大學九十五學年度碩士班招生試題
系所組別： 電子工程系碩士班乙一組
科　　目： 通信系統

總分共 100 分，作答時請於答案卷上標明題號

1. A random process *X(t)* is defined by $X(t) = A\cos(2\pi f_c t)$, where f_c is constant and A is a Gaussian-distributed random variable of zero mean and variance σ_A^2. The random process is applied to an ideal integrator, producing the output $Y(t) = \int_0^t X(\tau)d\tau$

   (a) (5%) Determine the probability density function of *Y(t)* at time t_k.
   (b) (5%) What is the definition of 'wide-sense stationary (WSS)'? Is *Y(t)* WSS?
   (c) (5%) What is the definition of 'ergodic process'? Is *Y(t)* ergodic?

2. Consider a narrowband noise $n(t)$ of bandwidth $2B$ centered on frequency f_c, as depicted in Fig. 1.

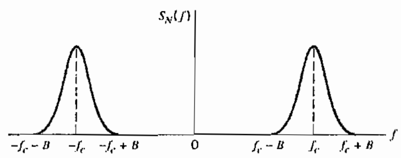

Fig. 1

   (a) (5%) Depict the functional block diagram of a noise analyzer, which is used to extract the in-phase component, $n_I(t)$, and the quadrature component, $n_Q(t)$, of $n(t)$.
   (b) (5%) Determine the power spectral densities of $n_I(t)$ and $n_Q(t)$ in terms of the power spectral density of $n(t)$. (P.S. Represents $S_{N_I}(f)$ and $S_{N_Q}(f)$ in terms of $S_N(f)$.)
   (c) (5%) Determine the cross-spectral densities of $n_I(t)$ and $n_Q(t)$.

3. A carrier wave of frequency 100 MHz is frequency-modulated by a sinusoidal wave of amplitude 20 volts and frequency 100 kHz. The frequency sensitivity of the modulator is 25 kHz per volt.

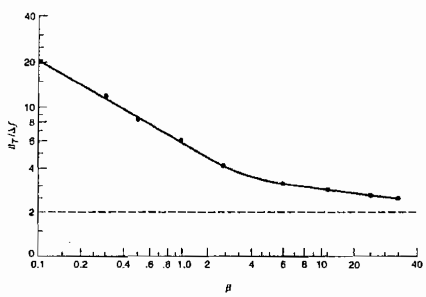

Fig. 2

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(a) (5%) Determine the approximate bandwidth of the FM signal, using Carson's rule.

(b) (5%) Determine the bandwidth by transmitting only those side frequencies whose amplitudes exceed 1 percent of the unmodulated carrier amplitude. Use the universal curve of Fig. 2 for this calculation.

(c) (5%) Repeat your calculations, assuming that the amplitude of the modulating signal is doubled.

(d) (5%) Repeat your calculations, assuming the modulation frequency is doubled.

4. Consider a binary baseband transmission with transmitted signals

$$s_0(t) = \begin{cases} A\cdot t, & 0 < t < T \\ 0, & \text{otherwise.} \end{cases}$$

and $s_1(t) = -s_0(t)$, where T is the bit duration and A is the peak value of $s_0(t)$. Assume that the signals are transmitted over an AWGN (additive white Gaussian noise) channel, where the two-sided power spectral density of the noise is $N_0/2$.

(a) (3%) Find the impulse response of the matched filter to $s_0(t)$ and $s_1(t)$.

(b) (3%) Under what sense is the matched filter optimal?

(c) (4%) What is the bit error rate in this binary transmission ? Please express your answer in terms of A, T, N_0, and the Q function, which is defined by

$$Q(z) \triangleq \int_z^\infty \frac{1}{\sqrt{2\pi}} e^{-x^2/2} dx.$$

5. Consider the set of signals below: $S = \{s_1(t) = A\cos(2\pi f_c t),\ s_2(t) = A\sin(2\pi(f_c + \frac{1}{T})t),\ s_3(t) = A\sin(2\pi f_c t),\ s_4(t) = -A\cos(2\pi f_c t);\ \text{for } 0 < t < T\}$, where T is the bit duration. They are to be chosen as the modulation waveforms for a digital communication. Assume that the communication is corrupted by an AWGN whose two-sided power spectral density is $N_0/2$.

(a) (2%) If we want to adopt BPSK (binary phase shift keying) modulation, which two signals in S should be chosen?

(b) (2%) If we want to adopt BFSK (binary frequency shift keying) modulation, which two signals in S should be chosen ?

(c) (3%) Plot a reception structure for the BPSK detection. Please use as few correlators as possible in it.

(d) (3%) Plot a noncoherent reception structure for the BFSK detection.

(e) (3%) The bandwidth efficiency is defined to be the ratio between the data rate (i.e. number of bits transmitted per second) and the consumed bandwidth (measured in Hz = 1/sec). Let us adopt the null-to-null bandwidth here. What is the bandwidth efficiency of BPSK modulation ?

(f) (4%) For $A = 4$, $T = 1$ and $N_0 = 1$, find the bit error rate in the BPSK detection.

<P.S.> You may need to use the Q function, which is defined and approximated as

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$$Q(z) \triangleq \int_{z}^{\infty} \frac{1}{\sqrt{2\pi}} e^{-x^2/2} dx, \text{ for } -\infty < z < \infty \approx \frac{1}{z\sqrt{2\pi}} e^{-z^2/2}, \text{ for } z \geq 3.$$

(g) (4%) For A = 4, T = 1 and N_0 = 1, find the bit error rate in the noncoherent BFSK detection.

6. In a 16-PSK communication system, the transmitted signals are

$$s_i(t) = \sqrt{E_s}\sqrt{\frac{2}{T_s}}\cos(2\pi f_c t + i \times \frac{\pi}{8}), 0 \leq t \leq T_s, i = 0,1,...,15.$$

Assume that the received signal is $r(t) = s_i(t) + n(t)$, where $n(t)$ is AWGN with a two-sided power spectral density $N_0/2$.

(a) (4%) Plot the constellation diagram of $\{s_i(t);\ i = 0, 1, ..., 15\}$. On this constellation diagram, show how 4-bit patterns should be assigned to its symbols so that the bit error rate can be made as small as possible.

(b) (3%) Continued from the previous subproblem, if the symbol error probability is 10^{-5}, what is the bit error rate?

7. The state transition diagram of a convolutional code is shown in Fig. 3:

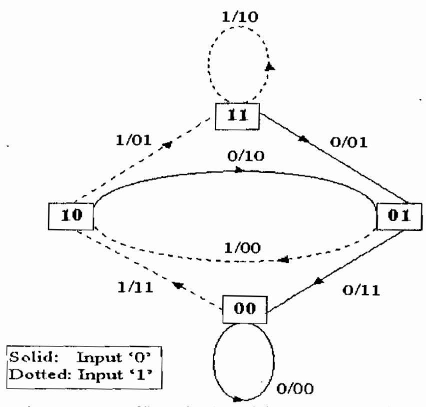

Fig. 3

where "$b / c_1 c_2$" on the branches indicates input message bit and output code bits.

(a) (3%) Find the codeword corresponding to the message word [1101001], assuming that the encoding begins at State 00.

(b) (4%) Plot the trellis diagram of this convolutional code up to four stages, assuming that the encoding begins at State 00.

(c) (2%) One popular method for decoding convolutional codes is the Viterbi algorithm. Does it perform the maximum a-posteriori estimation of the received word ?

(d) (3%) Under what condition regarding the communication channel is the Viterbi decoding with the Hamming metric equivalent to the maximum likelihood sequence estimation of the received word?

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