

國立台灣科技大學九十五學年度碩士班招生試題

系所組別：電子工程系碩士班乙二組

科目：工程數學

(一)請標題號並依題號順序作答，例如第一頁第一題，第二頁第二題，...等，

(二)總分 100 分。

$$1. A = \begin{bmatrix} 1 & 2 & 1 \\ 0 & -1 & 1 \\ 0 & 1 & 0 \end{bmatrix}, \text{ compute } A^8 - 2A^6 + 3A^4 + A^2 - 6I. (10\%)$$

2. (a) A is an $n \times n$ matrix. Suppose $A^k \bar{x} = 0$ has a vector solution $\bar{\alpha}$ and $A^{k-1} \bar{\alpha} \neq 0$, where k is an integer and \bar{x} is a vector. Is $\bar{\alpha}, A\bar{\alpha}, \dots, A^{k-1}\bar{\alpha}$ linearly independent or not? Prove your answer. (5%) (b) What values of a and b in the system

$$\begin{cases} x + 4y - 3z = 0 \\ 3x + 2y + z = 10b \\ y + az = -2 \end{cases} \text{ have (i) no solution, (ii) unique solution, and (iii) infinite solution.}$$

(5%)

$$3. A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & y & 1 \\ 0 & 0 & 1 & 2 \end{bmatrix} \text{ (a) if this matrix } A \text{ has one eigenvalue } 3, \text{ find } y. (5\%) \text{ (b) Diagonalize}$$

 $A^{-1}A. (10\%)$

4. A set of polynomials $\{f(x) = a_0 + a_1x + a_2x^2\}$, the possible basis function for the set

is $\{1, x, x^2\}$, (a) Applying Gram-Schmidt process to the basis functions to find theorthonormal basis functions if the inner product is defined as $\langle f(x), g(x) \rangle =$

$$\int_0^1 f(x)g(x)dx (10\%) \text{ (b) Representing } f(x) = 1 + x \text{ in terms of the orthonormal basis}$$

functions. (5%)

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Note: 答案卷內,請依題號順序作答

5. Solve $y' - e^y \cos(x) = 0$; $y(0) = 0$ (10%)

6.

(a). Show $y = -\ln|\csc(x)|$ is a solution of $y' = \cot(x)$ (5%)

(b). Solve

$$y'' + \frac{1}{x}y' + \left(1 - \frac{1}{4x^2}\right)y = 0 \quad \text{for } x > 0; \text{ where } y_1 = \frac{1}{\sqrt{x}}\sin(x) \text{ is a solution}$$

(10%)

7. Solve $y'' + 9y = 12 \sec(3x)$; (10%)

8. Solve $y'' + 3y' + 2y = 3\delta(x-2) + 2\delta(x-5)$, where $y'(0) = y(0) = 0$. (15%)

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