國立台灣科技大學九十五學年度碩士班招生試題

系所組別： 電子工程系碩士班乙三組

科　　目： 工程數學

總分 100 分

1. Briefly answer the following questions. You will not get any credit if only the answer is given.

   (a) (5 points) Consider a 3×3 system of linear equations $\mathbf{Ax} = \mathbf{b}$, where

   $$\mathbf{A} = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 5 & 8 \\ 3 & 5 & 7 \end{bmatrix} \quad \text{and} \quad \mathbf{b} = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$$

   Determine the condition on b_1, b_2, and b_3 such that $\mathbf{Ax} = \mathbf{b}$ does not have a solution.

   (b) (5 points) Let $\mathbf{A}$ be an $n \times n$ matrix with rank r, then which of the following matrices also has(have) rank r?

   $$3\mathbf{A}^T, \ \begin{bmatrix} 2\mathbf{A} & 3\mathbf{A} \end{bmatrix}, \ \begin{bmatrix} \mathbf{A} \\ \mathbf{A} \end{bmatrix}, \ \begin{bmatrix} \mathbf{A} & \mathbf{A} \\ \mathbf{A} & \mathbf{A} \end{bmatrix}$$

2. (5 points) Let $\mathbf{P}_n$ denote the set of all polynomials of degree less than n. Now, consider a subspace V of $\mathbf{P}_{10}$ which is given by

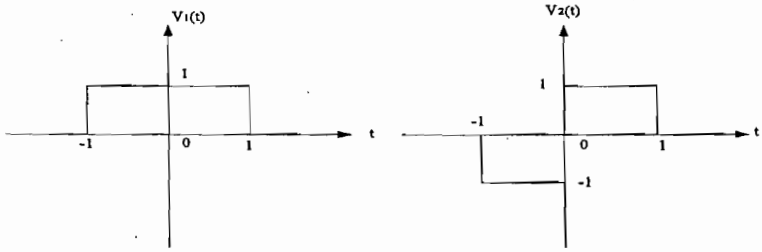
   $$\mathrm{V} = \{p(x) : p(x) = x^9 p(x^{-1})\}$$

   Determine dim(V).

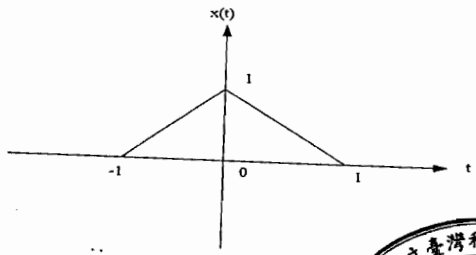
3. (a) (5 points) Suppose that $\mathbf{A} = \begin{bmatrix} 6 & -4 \\ \alpha & \beta \end{bmatrix}$, then determine α and β such that $\mathbf{A}$ has eigenvectors $\mathbf{x}_1 = \begin{bmatrix} 4 \\ 3 \end{bmatrix}$ and $\mathbf{x}_2 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$.

   (b) (5 points) Consider another 2×2 matrix $\mathbf{B}$ with the same eigenvectors $\mathbf{x}_1$ and $\mathbf{x}_2$ as (a) and with respective eigenvalues $\lambda_1 = 1$ and $\lambda_2 = 0$. Determine $\mathbf{B}^{10}$.

4. (10 points) Consider a communication system which transmits the message γ and η through a linear combination with two known waveforms $v_1(t)$ and $v_2(t)$ by $\gamma v_1(t) + \eta v_2(t)$, where γ and η are **real numbers**, and $v_1(t)$ and $v_2(t)$ are given by



   The receiver receives $x(t)$ and determines the transmitted γ and η by choosing γ and η which minimize $\left\| x(t) - (\gamma v_1(t) + \eta v_2(t)) \right\|$, where $\|y(t)\| = \sqrt{< y(t), y(t) >}$ with $< y(t), z(t) >= \int_{-1}^{1} y(t) z(t)\, dt$. Now suppose that the received signal $x(t)$ is as given below. Determine the transmitted γ and η.



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5. Consider the partial differential equation given by

$$\frac{\partial^2 \Phi(x,t)}{\partial x^2} = \eta^2 \frac{\partial^2 \Phi(x,t)}{\partial t^2}, \qquad 0 < x < a,\ t > 0$$

where η is a known constant.

(a) (7 points) Find a general solution for this partial differential equation.

(b) (8 points) Find the solution with initial condition

$$\Phi(0,t) = 0, \quad \Phi(a,t) = 0, \quad t > 0$$

$$\Phi(x,0) = 0, \frac{\partial \Phi(x,t)}{\partial t}\Big|_{t=0} = 1, \quad 0 < x < a$$

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6. Consider a differential equation of the form

$$y''(t) + 4y'(t) + 4y(t) = 2t + 1$$

with the initial conditions $y(0) = 0$ and $y(1) = 1$. Please find an explicit solution of this differential equation. (15 Points)

7. A complex function $f(z)$ is characterized by the formula $f(z) = f(x + iy) = u(x, y) + i\,v(x, y)$, where x and y are real-valued variables and $u(x, y)$ and $v(x, y)$ are real-valued functions. If $u(x, y) = x^3 - 3xy^2 + 2y$, please determine the general expression for $v(x, y)$ such that $f(z)$ is analytic inside the unit circle on the complex plane. (10 Points)

8. Suppose n is a positive integer. Please determine all roots of the equation

$$[(z-1)^n - 1]\,[(z-1)^{3n} + (z-1)^{2n} + (z-1)^n + 1] = 0.$$

(10 Points)

9. A complex function $f(z)$ is defined by $f(z) = \frac{e^z}{(z^4+0.5z^3)}$. (a) Please determine residue of $f(z)$ at $z = 0$. (5 Points) (b) Please find residue of $f(z)$ at $z = -0.5$. (5 Points) (c) Please find $\oint_C f(z)dz$, where C is a closed counterclockwise contour on the unit circle. (5 Points)