國立台灣科技大學九十五學年度碩士班招生試題

系所組別： 電子工程系碩士班丙組

科　　目： 工程數學

(一)請標題號並依題號順序做答，例如第一頁第一題，第二頁第二題，...等，(二)總分 100 分。

1. $A=\begin{bmatrix} 1 & 0 & 0 & 0 \\ -2 & 3 & 0 & 0 \\ 0 & -4 & 5 & 0 \\ 0 & 0 & -6 & 7 \end{bmatrix}$ and $B=(I_{4\times4}+A)^{-1}(I_{4\times4}-A)$. Find $(I_{4\times4}+B)^{-1}$. ***(10%)***

2. (a)Changing the basis $\{1,x,x^2\}$ to an orthonormal basis by using Gram-Schmidt process and the inner product is defined as $\langle p(x),q(x)\rangle=\sum_{i=1}^{3}p(x_i)q(x_i)$ where $x_1=-1$, $x_2=0$, $x_3=1$. ***(10%)*** (b) Representing 1+x in terms of the orthonormal basis. ***(5%)***.

3. (a) Matrix A has characteristic polynomial $f_A(\lambda)=(\lambda-\lambda_1)^{m_1}(\lambda-\lambda_2)^{m_2}$. On what condition A can be diagonalized. ***(5%)*** (b) $A=\begin{bmatrix} 0 & 0 & -2 \\ 0 & 1 & 0 \\ 1 & 0 & 3 \end{bmatrix}$ has eigenvalues 1, 1, 2. Diagonalize A. ***(10%)***.

4. Matrix D_n is denoted by $\begin{bmatrix} a_1+b_1 & a_1+b_2 & \cdots & a_1+b_n \\ a_2+b_1 & a_2+b_2 & \cdots & a_2+b_n \\ \vdots & \vdots & & \vdots \\ a_n+b_1 & a_n+b_2 & \cdots & a_n+b_n \end{bmatrix}$. Find the determinant of D_n, if n = 1, n = 2, and $n\geq 3$. ***(10%)***(Hint: Decompose D_n into a product of two matrices)

國立台灣科技大學九十五學年度碩士班招生試題

系所組別： 電子工程系碩士班丙組

科　　目： 工程數學

Note: 答案卷內,請依題號順序作答

5. Solve $y'-e^{-y}\cos(x)=0$; $y(0)=0$ (10%)

6.

(a).Show $y=-\ln|\csc(x)|$ is a solution of $y'=\cot(x)$ (5%)

(b).Solve

$$y''+\frac{1}{x}y'+\left(1-\frac{1}{4x^2}\right)y=0 \quad for \quad x>0;\ where \quad y_1=\frac{1}{\sqrt{x}}\sin(x) \quad is \quad a \quad solution.$$

(10%)

7. Solve $y''+9y=12\sec(3x)$; (10%)

8. Let's consider the following heat equation and its boundary condition

$$\frac{\partial u}{\partial t}=k\frac{\partial^2 u}{\partial x^2} \quad for \quad 0<x<L, t>0,$$

$$u(0,t)=u(L,t)=0 \quad for \quad t\geq 0,$$

$$u(x,0)=f(x) \quad for \quad 0\leq x\leq L$$

Let $u(x,t)=X(x)T(t)$. Solve $u(t)$. (15%)