國立台灣科技大學九十五學年度碩士班招生試題

系所組別：電機工程系碩士班乙一組

科　　目：微分方程及線性代數

總分 100 分

1. (15 points) Find y(t), which satisfies the given function and initial conditions, by Laplace transform.

$$y'' + 4y' + 4y = 1 + \delta(t-1), y(0) = 0, y'(0) = 0.5$$

2. (10 points) Find a general solution to the differential equation. express y explicitly as a function of x.

$$y'' - 4y = 8x^2 - 2x$$

3. (10 points) Find a general solution to the ordinary differential equation

$$y'' + 2y' + y = \frac{1}{t^3 e^t} + 3$$

4. (15 points) Find the solution of the differential equation satisfying the initial condition $y(0) = 3$, express y explicitly as a function of x.

$$\frac{dy}{dx} = \frac{xy^3}{x^2+1}$$

5. (20 points; 10 points each) Let $T : \mathbb{R}^2 \to \mathbb{R}^2$ be the transformation that rotates each point in $\mathbb{R}^2$ about the origin through an angle φ, with counterclockwise rotation for a positive angle.
   (a) Find the standard matrix of this transformation A.
   (b) Given a point $(1, -1) \in \mathbb{R}^2$, find the result of the transformation by T when $\varphi = 45^o$.

6. (10 points) Find an LU factorization of

$$A = \begin{bmatrix} 2 & 4 & -1 & 5 & -2 \\ -4 & -5 & 3 & -8 & 1 \\ 2 & -5 & -4 & 1 & 8 \\ -6 & 0 & 7 & -3 & 1 \end{bmatrix}$$

7. (10 points) Let $b_1 = \begin{bmatrix} -9 \\ 1 \end{bmatrix}$, $b_2 = \begin{bmatrix} -5 \\ -1 \end{bmatrix}$, $c_1 = \begin{bmatrix} 1 \\ -4 \end{bmatrix}$, $c_2 = \begin{bmatrix} 3 \\ -5 \end{bmatrix}$, and consider the bases for $\mathbb{R}^2$ given by $B = \{b_1, b_2\}$ and $C = \{c_1, c_2\}$. Find the change-of-coordinates matrix from B to C.

8. (10 points) Diagonalize the following matrix, if possible.

$$A = \begin{bmatrix} 2 & 4 & 3 \\ -4 & -6 & -3 \\ 3 & 3 & 1 \end{bmatrix}$$

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