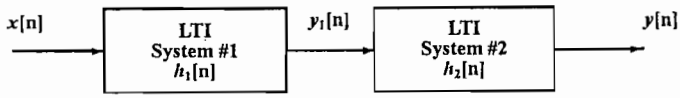
國立台灣科技大學九十五學年度碩士班招生試題

系所組別： 電機工程系碩士班乙一組

科　　目： 信號與系統

總分 100 分，請依序作答。

1. For a cascaded system



Suppose that system #1 is a filter described by the difference equation

$$y_1[n] = \sum_{k=0}^{6} \alpha^k x[n-k]$$

and the system #2 is described by the impulse response

$$h_2[n] = \delta[n] - \alpha\delta[n-1]$$

where α is a real number

(a) Determine the impulse response h[n] of the overall system. (10 %)

(b) Find the difference equation that relates y[n] to x[n] for $\alpha = 0.5$. (5%)

2. For a periodic signal

$$x(t) = 1 + 2\cos(300\pi t + 0.25\pi) + \sin(500\pi t)$$

(a) What is the fundamental period T of x(t)? (5%)

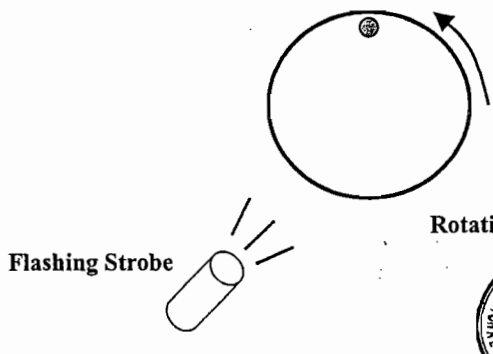
(b) Determine the Fourier series coefficients a_k of x(t). (5%)

If there is another periodic signal $y(t) = 10 + 5\cos(300\pi t)$

(c) Determine the Fourier series coefficients b_k of w(t) where w(t)=x(t)y(t). (5%)

(d) Determine the Fourier series coefficients c_k of z(t) where z(t) is the periodic convolution of x(t) and y(t), that is $z(t) = \int_T x(\tau)y(t-\tau)d\tau$ (5%)

3. In the rotating disk and strobe demo, a disk with a single spot marked on it is rotating counterclockwise at a constant rate (f_r revolutions per second). The position of the spot is sampled by a flashing strobe at a periodic rate (f_s Hz). We can observe that different flashing rates of the strobe light would make the spot on the disk stand still or move in different directions.



116

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(a) f_r = 12 revolutions per second. f_s = 14 Hz, express the movement of the spot on the disk as a complex phasor, p[n], that gives the position of the spot at the n-th flash. Assume that the spot is at the top when $n = 0$ (the first flash). (5%)

(b) For the conditions in part (a), determine the apparent speed (in revolutions per second) and direction of movement of the "strobed" spot. (5%)

(c) Now assume that f_r is unknown and f_s = 17 Hz. The spot on the disk moves clockwise by 20 degrees with each flash, determine f_r (in revolutions per second). Give all possible solutions. (5%)

4. A continuous-time signal $x(t)$ has Fourier transform

$$X(\omega)=\frac{k}{\alpha+j\omega}, \qquad \alpha>0$$

where k and α are constants. Determine the Fourier transform $Y(\omega)$ of the following signals.

(a) $y(t)=t\cdot x(t)$ (5%)

(b) $y(t)=x^2(t)$ (5%)

(c) $y(t)=\frac{1}{\alpha-jt}$ (5%)

5. A continuous-time system is given by the input/output differential equation

$$\frac{d^2y(t)}{dt^2}+a\frac{dy(t-1)}{dt}-y(t)=\frac{dx(t)}{dt}+x(t-2)$$

(a) Find the transfer function $H(s)$ of the system. (5%)

(b) Let $v(t)=L^{-1}(\frac{1}{s^2+ase^{-s}-1})$, derive the expression for the impulse response $h(t)$ of the system in terms of $v(t)$. (10%)

(c) If a=0, and $y(0^-)=0, \dot{y}(0^-)=1, x(t)=u(t)$, computer the response y(t) for all $t\geq 0$. (10%)

6. Compute the inverse z-transform $x[n]$ of the following transforms. Determine $x[n]$ for all $n\geq 0$.

(a) $X(z)=\frac{z^2-1}{z^2+1}$, $|z|>1$ (5%)

(b) $X(z)=\ln(\frac{2z-1}{2z})$, $|z|>0.5$ (5%)