

國立台灣科技大學九十五學年度碩士班招生試題

系所組別：資訊工程系碩士班

科目：計算機數學

(總分：100 分)

1. (10%) February 29 occurs only in leap years. Years divisible by 4, but not by 100, are always leap years. Years divisible by 100, but not by 400, are not leap years, but years divisible by 400 are leap years. What probability distribution for birthdays should be used to reflect how often February 29 occurs?
2. (10%) Show that the system of congruences $x \equiv a_1 \pmod{m_1}$ and $x \equiv a_2 \pmod{m_2}$ has a solution if and only if $\gcd(m_1, m_2) \mid a_1 - a_2$.
3. In how many ways can two dozen identical robots be assigned to four assembly lines with
 - (a) (6%) at least three robots assigned to each line?
 - (b) (6%) at least three, but no more than nine, robots assigned to each line?
4. (a) (5%) For the finite state machine given in Table 1, determine a minimal machine that is equivalent to it.
 (b) (5%) Find a minimal string that distinguishes states s_4 and s_6 .

Table 1

| | v | | w | |
|-------|-------|-------|---|---|
| | 0 | 1 | 0 | 1 |
| s_1 | s_7 | s_6 | 1 | 0 |
| s_2 | s_7 | s_7 | 0 | 0 |
| s_3 | s_7 | s_2 | 1 | 0 |
| s_4 | s_2 | s_3 | 0 | 0 |
| s_5 | s_3 | s_7 | 0 | 0 |
| s_6 | s_4 | s_1 | 0 | 0 |
| s_7 | s_3 | s_5 | 1 | 0 |
| s_8 | s_7 | s_3 | 0 | 0 |

5. For the graph G in Figure 1, answer the following questions and give the reasons.

- (a) (4%) Does G have an Euler circuit or a Hamilton cycle?
- (b) (4%) Is G bipartite? Is it planar?

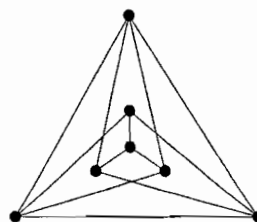


Figure 1

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6. Let $A = \begin{pmatrix} 1 & 4 & -2 \\ -3 & 4 & 0 \\ -3 & 1 & 3 \end{pmatrix}$.

(a) (4%) Find a matrix P that diagonalizes A .

(b) (3%) Find $P^{-1}AP$.

(c) (3%) Find A^4 .

7. (10%) Let $T: R^n \rightarrow R^n$ be a linear transformation. Prove that T is one-to-one if and only if T is onto.

8. (10%) Let A and B be $n \times n$ matrices. Prove that $\text{tr}(AB) = \text{tr}(BA)$.

9. (a) (6%) Let A and B be $n \times n$ matrices such that AB is invertible. Prove that both A and B are invertible.

(b) (4%) If one of A and B is not a square matrix, does the invertibility of AB imply that both A and B are invertible? Explain your answer.

10. (10%) Let A and B be $n \times n$ matrices. Prove that AB and BA have the same set of eigenvalues.

